RUTA: a framework for assessing and selecting additive value functions on the basis of rank related requirements

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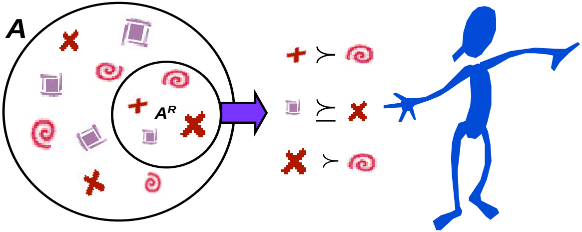
Outline

1. Indirect preference information
   for multiple criteria ranking and sorting
2. Modeling of rank related requirements
3. Exploitation of the set of compatible value functions
   - Selection of a single value function
   - The necessary and the possible
   - Extreme ranking analysis
4. Illustrative example
5. Selection of a representative value function
   for extreme ranking analysis
6. Summary

Preference disaggregation for multiple criteria ranking

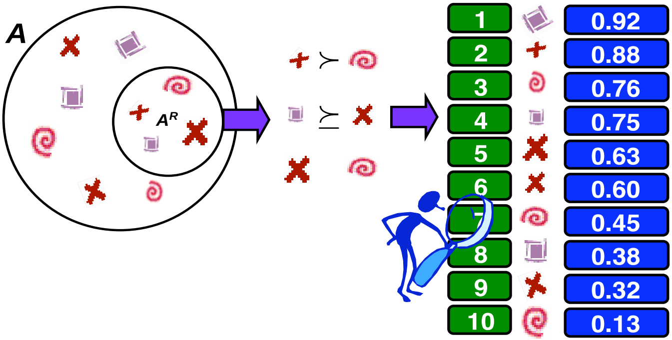
multiple criteria ranking



pairwise comparisons

- Find mathematical model reproducing exemplary decisions
- Inferring compatible instances from complete or partial preorder
- Consistent with intuitive reasoning of DM
- Example: the UTA method and its several variants

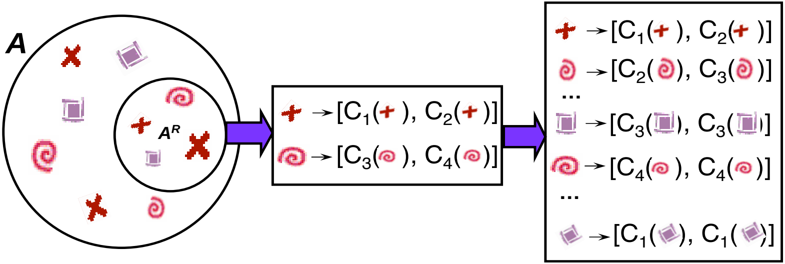
Complete rankings and interest of the DM



- Focus on complete rankings: intuitive, easy to understand, and popular
- Example: comprehensive values or net outranking flows
- Interest in the positions attained by alternatives and their comprehensive scores

Preference disaggregation for multiple criteria sorting

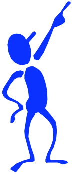
multiple criteria sorting



assignment examples

- Inferring compatible instances of the preference model from assignment examples
- DM refers to desired final recommendation (assignment) for reference alternatives
- Example: UTADIS and ELECTRE TRI (disaggregation approach)

Are desired ranks really used in the judgments?



should take place **on the podium**

should (not) be ranked **among top / bottom 5 alternatives**

should be among **the 10% of best / worst alternatives**

is predisposed to secure **the place between 4 and 10**

should be ranked **in the second ten of alternatives**

should be ranked **in the upper / lower half of the ranking**

evaluation profile of predisposes it **to have value at least / at most *x***

where $x \in [0,1]$

- People are used to refer to desired ranks of alternatives
- Range of allowed ranks that alternative should attain
- Rate a given alternative individually
  and collate it with all remaining alternatives
- Desired scores: set of benchmarks

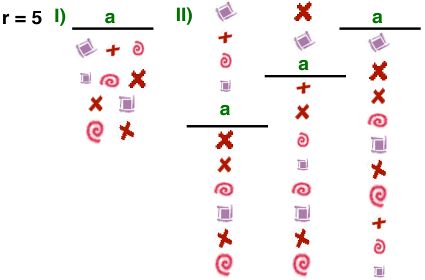
Modeling of rank related requirements

a should be ranked **among top r alternatives**

I) $U(a) + M \cdot v^{>}_{a,b} \geq U(b) + \varepsilon, \quad for\ all\ b \in A \setminus \{a\}$

II) $\sum_{b \in A \setminus \{a\}} v^{>}_{a,b} \leq r - 1$

example

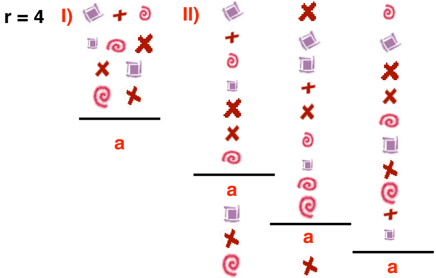


a should be ranked **among bottom r alternatives**

I) $U(b) + M \cdot v^{<}_{a,b} \geq U(a) + \varepsilon, \quad for\ all\ b \in A \setminus \{a\}$

II) $\sum_{b \in A \setminus \{a\}} v^{<}_{a,b} \leq r - 1$

example



- Alternative a starts with rank "1" for "top" case ("n" for "bottom")
- If $v_{a,b} = 1$, corresponding constraint is always satisfied
- Such relaxation is admitted for up to $r - 1$ constraints

Modeling of rank related requirements

a should be ranked **among top *p%* alternatives**

I) $U(a) + M \cdot v_{a,b}^{>} \geq U(b) + \varepsilon, \quad for\ all\ b \in A \setminus \{a\}$

II) $\sum_{b \in A \setminus \{a\}} v_{a,b}^{>} \leq \lfloor n \cdot p\% \rfloor - 1$

a should be ranked **among bottom *p%* alternatives**

I) $U(b) + M \cdot v_{a,b}^{<} \geq U(a) + \varepsilon, \quad for\ all\ b \in A \setminus \{a\}$

II) $\sum_{b \in A \setminus \{a\}} v_{a,b}^{<} \leq \lfloor n \cdot p\% \rfloor - 1$

5% **10%** **a quarter** **one third** **half**

- Modeled analogously to situation when the number of alternatives is provided explicitly
- If $p\%$ of n is not equal to a natural number, we need to refer to $\lfloor n \cdot p\% \rfloor$

Modeling of rank related requirements

a should be ranked **in a position in the range [f,c], with f ≤ c**

I) $U(a) + M \cdot v^{>}_{a,b} \geq U(b) + \varepsilon, \quad for\ all\ b \in A \setminus \{a\}$

II) $\sum_{b \in A \setminus \{a\}} v^{>}_{a,b} \leq c - 1$

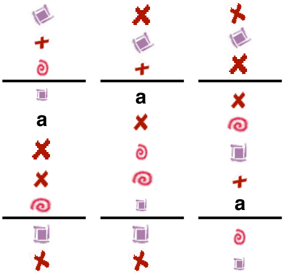
III) $U(b) + M \cdot v^{<}_{a,b} \geq U(a) + \varepsilon, \quad for\ all\ b \in A \setminus \{a\}$

IV) $\sum_{b \in A \setminus \{a\}} v^{<}_{a,b} \leq n - f$

V) $v^{>}_{a,b} + v^{<}_{a,b} \leq 1, \quad for\ all\ b \in A \setminus \{a\}$

example

[f,c] = [4,8]



- The most general statement
- At most $c - 1$ alternatives ranked not worse than *a*
- At most $n - f$ alternatives ranked not better than *a*
- Subsets of alternatives which are ranked better or worse than *a* are disjoint

Pairwise comparisons as rank related requirements

a should be ranked **higher** than b

$$U(a) \geq U(b) + \varepsilon$$

a should be ranked **not lower** than b

$$U(a) \geq U(b)$$

a should be ranked **the same** as b

$$U(a) = U(b)$$

- Combine rank related requirements with "traditional" pairwise comparisons
- Perceive statements: "*a* is (weakly) preferred to *b*" or "*a* is indifferent to *b*" in a slightly different way

Set of compatible value functions

$$U(a) + M \cdot v^{>}_{a,b} \geq U(b) + \varepsilon, \quad for\ all\ b \in A \setminus \{a\}$$

$$\sum_{b \in A \setminus \{a\}} v^{>}_{a,b} \leq P_{*,DM}(a) - 1$$

$$U(b) + M \cdot v^{<}_{a,b} \geq U(a) + \varepsilon, \quad for\ all\ b \in A \setminus \{a\}$$

$$\sum_{b \in A \setminus \{a\}} v^{<}_{a,b} \leq n - P^{*,DM}(a)$$

$$v^{>}_{a,b} + v^{<}_{a,b} \leq 1, \quad for\ all\ b \in A \setminus \{a\}$$

if $a \in A^R$ should be ranked in the range $[P^{*,DM}(a), P_{*,DM}(a)]$

$$U(a) \in [U_{*,DM}(a), U^{*,DM}(a)]$$

if there are requirements with respect to U(a) for $a \in A^R$

$$U(a) \geq U(b), \quad U(a) = U(b), \quad U(a) \geq U(b) + \varepsilon$$

if there are some „traditional" pairwise comparisons

$$u_j(x_j^k) - u_j(x_j^{k-1}) \geq \kappa = 0, \quad j \in J, k = 2, \ldots, n_j(A)$$

monotonicity

$$u(x_j^1) = 0, \quad j \in J, \quad \sum_{j=1}^{m} u_j\left(x_j^{n_j(A)}\right) = 1$$

normalization

If $E(A^R)$ is feasible and $\varepsilon^* = max\ \varepsilon$, subject to $E(A^R)$ is greater than 0, there is at least one compatible value function

Selection of a single value function

I) Maximize : ε

based on UTAMP1

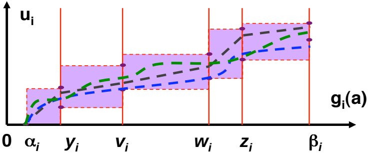
II) Maximize : $\varepsilon + \kappa$

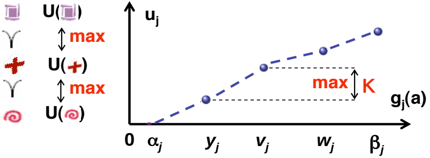
based on UTAMP2

III) Maximize:

$\sum \varepsilon(a,b) + \sum_j \sum_k \kappa(k,j)$

based on ACUTA





s.t. constraints defining the set of compatible value functions $E(A^R)$

- Existing UTA-like techniques, supposing preference information in form of pairwise comparisons
- Easy adaptation to use in the context of rank related requirements
- UTAMP1, UTAMP2, ACUTA, REPRESENTATIVE

Robust ordinal regression

the necessary $\succsim^N$ vs $\succsim^P$ the possible

for all U in U^R : U(a) ≥ U(b)

for at least one U in U^R : U(a) ≥ U(b)

prove that

U(b) > U(a) is impossible ✗

✓ U(a) ≥ U(b) is possible

i.e., solve the following MILP

$$\text{Maximize}: \varepsilon \quad \begin{cases} U(b) \geq U(a) + \varepsilon \\ E(A^R) \end{cases}$$ ✗

✓ $$\text{Maximize}: \varepsilon \quad \begin{cases} U(a) \geq U(b) \\ E(A^R) \end{cases}$$

draw conclusion on the basis of the optimal value of ε

if max ε ≤ 0
or infeasible set of constraints
a is necessarily preferred to b

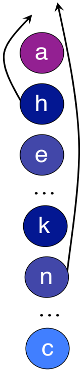
if max ε > 0
and feasible set of constraints
a is possibly preferred to b

always, not always

sometimes, never

- Apply all compatible value functions
- Indicate alternatives which are ranked better or worse than *a*

Extreme ranking analysis



the highest $P^*(a)$ rank $P_*(a)$ the lowest

assume that *a* is ranked

in the top | in the bottom

identify minimal subset of alternatives that are simultaneously

not worse than a | not better than a

i.e., solve the following MILP

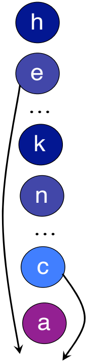
$$Minimize: f_{\max}^{pos} = \sum_{b \in A \setminus \{a\}} v_b \qquad Minimize: f_{\min}^{pos} = \sum_{b \in A \setminus \{a\}} v_b$$

$$\begin{cases} U(a) > U(b) - M \cdot v_b, \ for \ all \ b \in A \setminus \{a\} \\ E(A^R) \end{cases} \qquad \left. \begin{matrix} U(b) > U(a) - M \cdot v_b \\ E(A^R) \end{matrix} \right\}$$

read off the extreme ranks

$$P^*(a) = f_{\max}^{pos} + 1 \qquad P_*(a) = |A| - f_{\min}^{pos}$$

the best case | the worst case



- Assuming that *a* is in top 3, what is the best rank of *b*?
- Interval orders using ranges of possible ranks

Illustrative example

Denmark
Slovakia
Sweden
Poland
UK
Bulgaria
Netherlan
Turkey
Norway
Russia
Malta
Kazakh.
France
Ukraine
Germany
Azerbaij.
Ireland

connectivity
business environment
social and cultural environment
legal environment
government policy and vision
consumer and business adoption

Sweden	top 3
Netherlan.	4 - 5
France	second 5
Bulgaria	outside top 10
Turkey	bottom 5

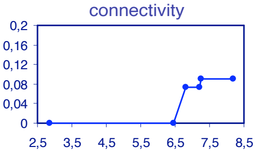
- Digital economy (Economist Intelligence Unit in 2010)
- Quality of information and technology infrastructure
- 17 European countries evaluated on 6 criteria

Selection of a single value function

Ranking and comprehensive values

Denmark	1	1,000	Slovakia	10	0,417
Sweden	1	0,917	Poland	10	0,417
UK	3	0,833	Bulgaria	12	0,333
Netherlan	4	0,750	Turkey	13	0,250
Norway	5	0,667	Russia	14	0,166
Malta	6	0,625	Kazakh.	15	0,083
France	7	0,583	Ukraine	15	0,083
Germany	8	0,500	Azerbaij.	17	0,000
Ireland	8	0,500			

Marginal value functions



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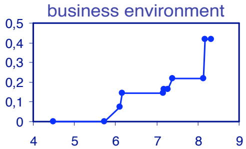
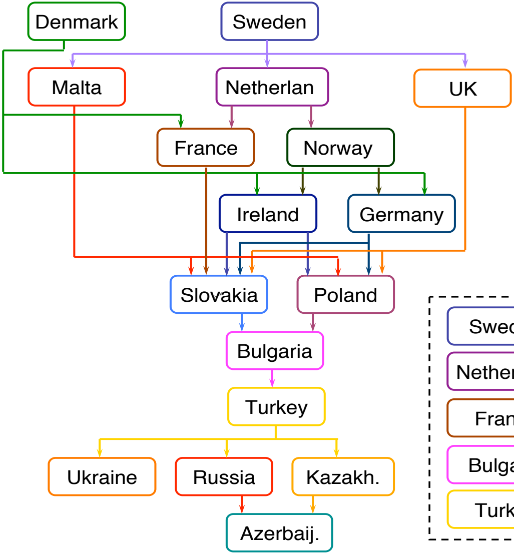
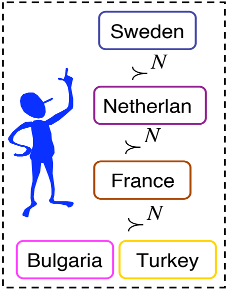


- Adaptation of UTAMP1 - maximize the minimal difference ε
- Reference alternatives placed at positions which are in the range of desired ranks
- Desired “parts” of final ranking

Robust ordinal regression





Sweden	top 3
Netherlan.	4 - 5
France	second 5
Bulgaria	outside top 10
Turkey	bottom 5

Extreme ranking analysis

Sweden	1	2
Denmark	1	5
UK	3	9
Netherlan	4	5
Malta	2	9
Norway	5	7
France	6	9
Germany	6	9
Ireland	6	9
Slovakia	10	11
Poland	10	11
Bulgaria	12	12
Turkey	13	13
Russia	14	16
Kazakh.	14	16
Ukraine	14	17
Azerbaij.	16	17

$P^{*}(a) \geq P^{*,DM}(a)$

$P_{*}(a) \leq P_{*,DM}(a)$

Sweden	top 3
Netherlan.	4 - 5
France	second 5
Bulgaria	outside top 10
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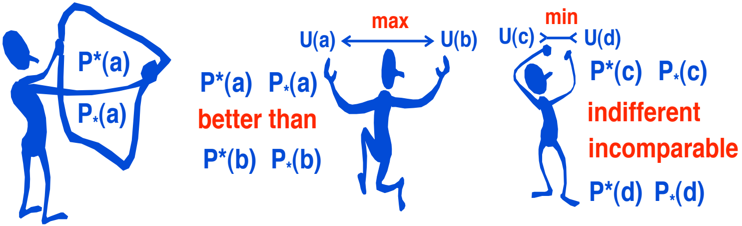
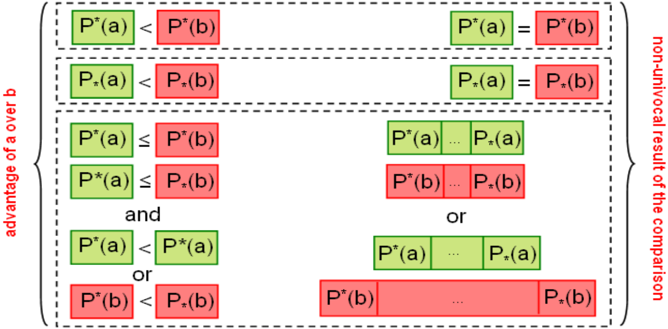
- Actual range of attained ranks is a proper subset of the range specified by the DM
- Evaluation profile vs. width of the range of possible ranks

Representative function for extreme ranking analysis



- “Flatten” consequences of applying all compatible instances
- See a score, assess relative importance of the criteria
- Built on results of extreme ranking analysis
- Pre-defined targets concerning enhancement of differences between scores of pairs of alternatives
- The DM is left the freedom of assigning priorities to different targets

Representative function for extreme ranking analysis



- $P^*(a)$ and $P_*(a)$ - the best and the worst rank attained by a in the set of all compatible value functions
- Emphasize the advantage of some alternatives over the others
- Reduce the ambiguity in the statement of the advantage
- Conditions with different intensity

Why not a pre-defined rule?

American football

highly specialized players

$P^*(a)$ $P^*(b)$

Futsal

universal players

$P_*(a)$ $P_*(b)$

- Good for at least one compatible value function
- Good for all compatible value functions

Summary

- Assessing and selecting additive value functions on the basis of rank related requirements
- RUTA: new preference disaggregation method
- Motivation: common use of rank related requirements and disadvantages of rankings obtained with traditional UTA-like methods
- Mixed-integer programming models
- Existing UTA-like procedures remain valid
- Selection of a representative value function for extreme ranking analysis