Modelling interactions on bipolar scales using robust ordinal regression: the UTAGSS method

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Plan

- Basic idea of bipolar interactions
- Technical discussion
- Didactic example
- UTA^{GSS} vs. UTS^{GMS}-INT
- Conclusions

Bipolar interactions

Illustrative example: positive (non-bipolar) interaction with respect to Mathematics and Literature

Students	Mathematics	Physics	Literature
S1	Good	Medium	Bad
S2	Good	Bad	Medium
S3	Medium	Medium	Bad
S4	Medium	Bad	Medium

$$S2 \succ S1 \quad \text{and} \quad S3 \succ S4$$

Illustrative example: positive (non-bipolar) interaction with respect to Mathematics and Literature

Students	Mathematics	Physics	Literature
S1	~~Good~~	Medium	Bad
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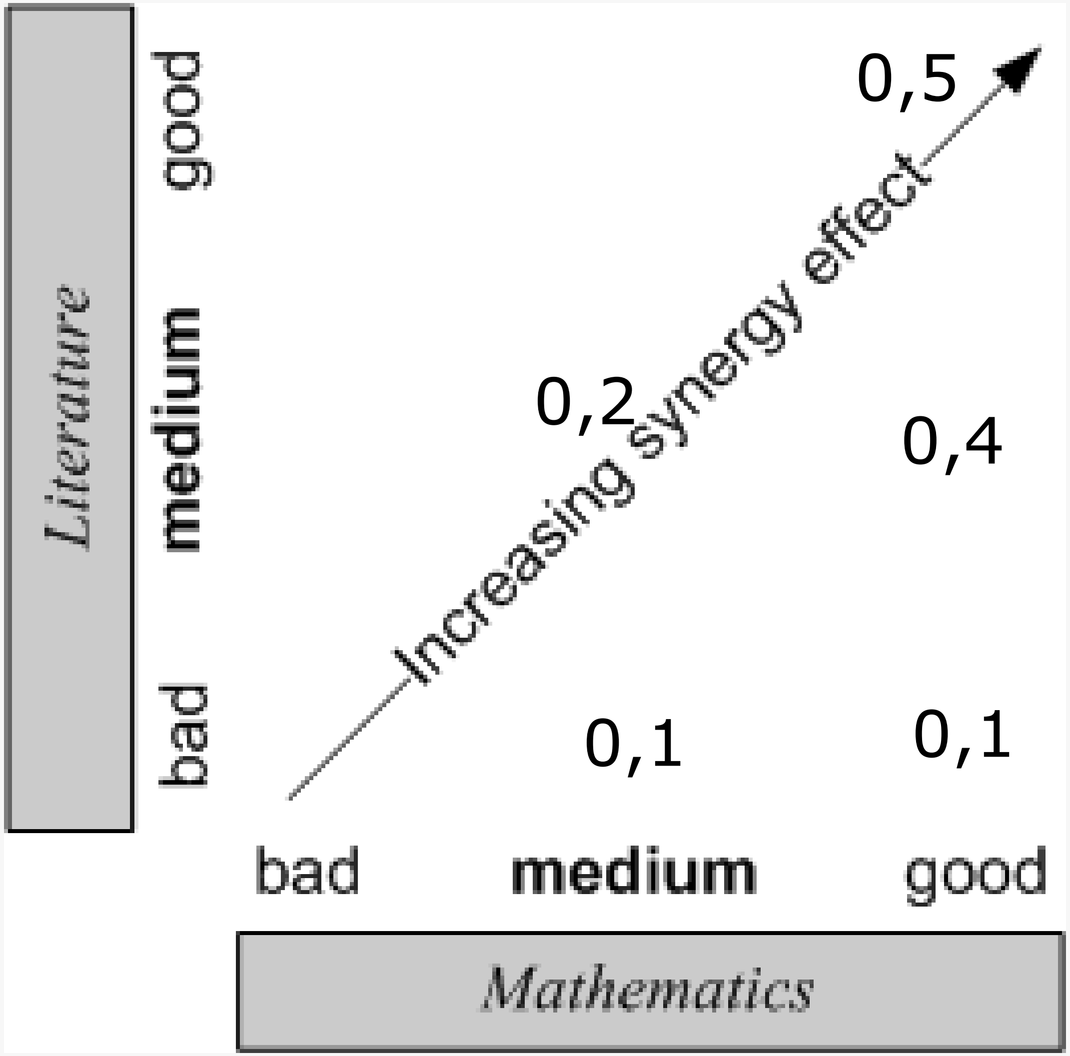
$$S2 \succ S1 \quad \text{and} \quad S3 \succ S4$$

Illustrative example: positive (non-bipolar) interaction with respect to Mathematics and Literature

Students	Mathe-matics	Physics	Litera-ture	Synergy (Math,Lit)	Global Score
S1	Good 0.3	Medium 0.3	Bad 0	Good, Bad 0.1	**0.7**
S2	Good 0.3	Bad 0	Medium 0.1	Good, Medium 0.4	**0.8**
S3	Medium 0.2	Medium 0.3	Bad 0	Medium, Bad 0.1	**0.6**
S4	Medium 0.2	Bad 0	Medium 0.1	Medium,Medium 0.2	**0.5**

$S2 \succ S1$ and $S3 \succ S4$

Illustrative example: positive (non-bipolar) interaction with respect to Mathematics and Literature



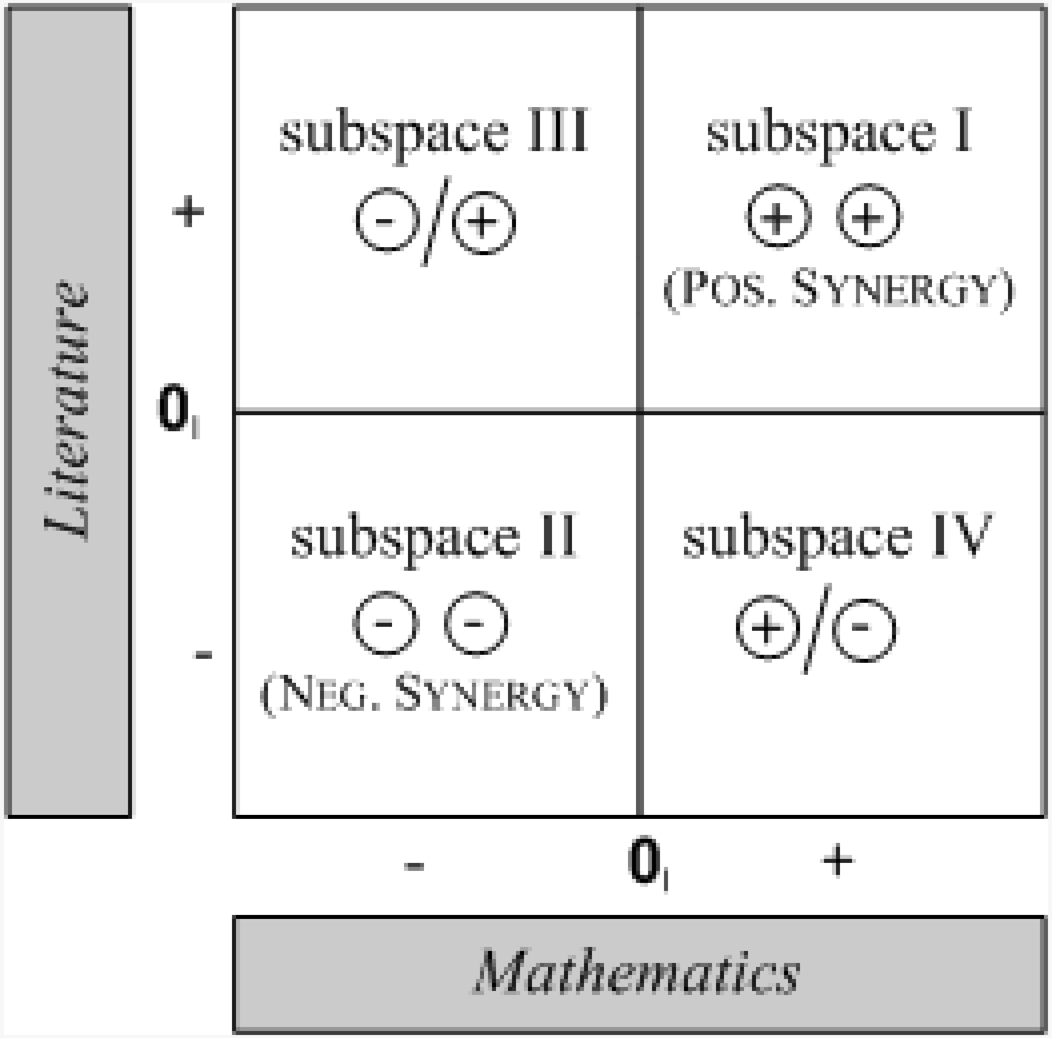
Bipolarity

- **Psychological experiments** as well as **everyday experiences** show that DMs are affected simultaneously by positive and negative feelings. (Cacioppo and Berntson, 1994; Caicioppo et al. 1997; Dubois, Fargier, and Bonnefon 2008; Osgood, Suci, and Tannenbaum, 1957; Slovic, Finucane, Peters, and MacGregor, 2002).
- When **choosing a movie**, the presence of a good actress is a positive argument; a noisy theater or bad critiques are negative arguments, but all three arguments are considered simultaneously. (Dubois, Fragier, Bonnefon 2008).
- **Cumulative Prospect Theory** (Tversky and Kahneman, 1979) attempts explicitly to account for positive and negative arguments numerically by proposing to compute a “net predisposition” of a decision, as the difference between functions of the two sets (termed “capacities”).

Bipolarity

- The **constitutive element** of a bipolar scale is a **reference point** that separates positive from negative performances and represents neutrality.
- Therefore this reference point is called neutral level $\mathbf{0}_i$**;** the subscript *i* denotes the criterion. The **neutral level** divides the positive and negative levels of the evaluated criterion. (Dubois, Fragier, and Bonnefon, 2008).
- For the existence of such a neutral level it is necessary that opposite notions of common language is used, for instance attractiveness and repulsiveness.
- In the above student example the evaluation ‘Medium’ can be interpreted as neutral level, ‘Good’ as positive, and ‘Bad’ as negative (Labreuche and Grabisch, 2006).

Illustrative example: positive **bipolar** interactions with respect to Mathematics and Literature



Technical discussion

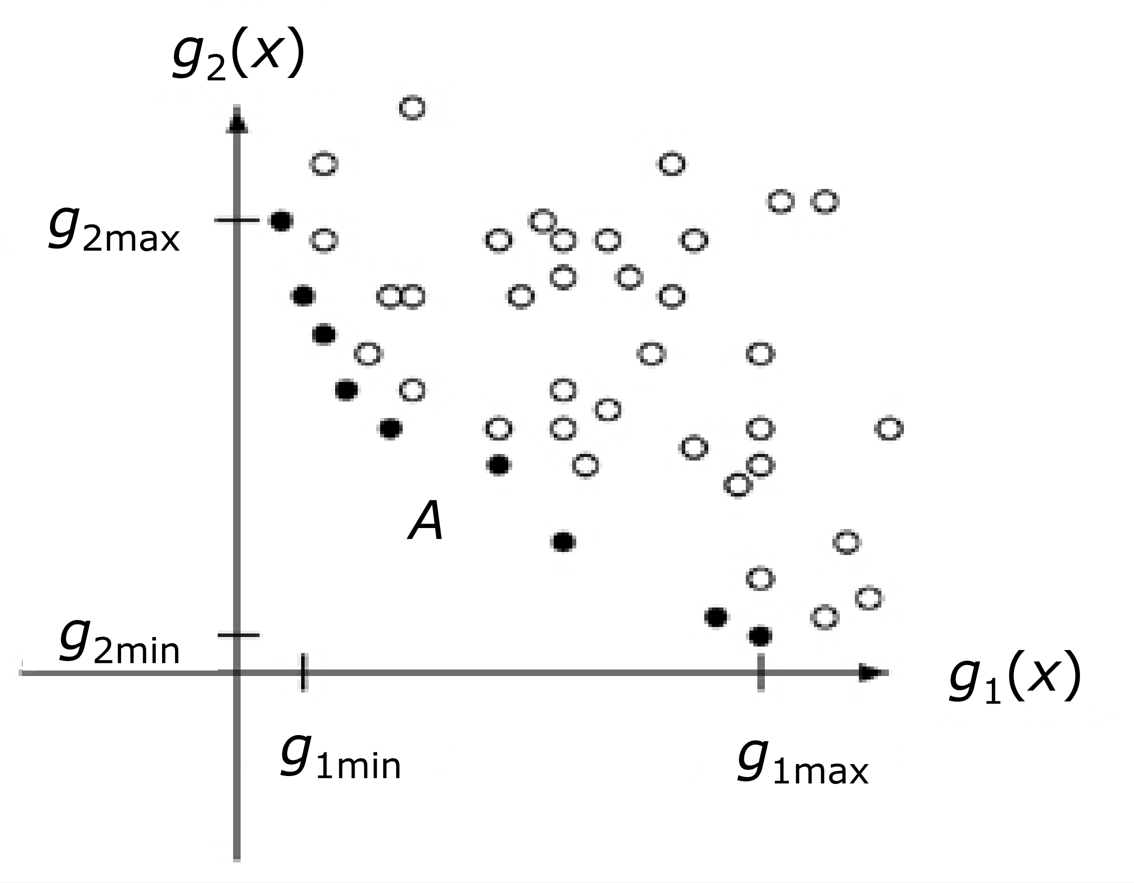
Plan of technical discussion

- Ordinal regression
- Robust ordinal regression
- The UTA^{GMS} method
- From UTA^{GMS}–INT method to the UTA^{GSS} method

Ordinal regression

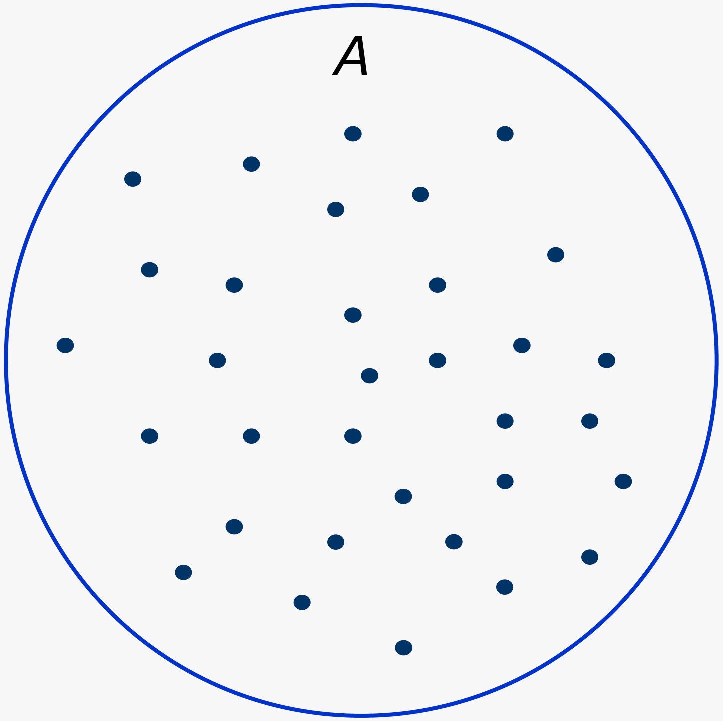
Problem statement – multicriteria choice, ranking and sorting

- Consider a finite set A of actions (actions, solutions, objects) evaluated by m criteria from a consistent family $F=\{g_1,...,g_m\}$; $I=\{1,\ldots,m\}$.
- The only objective information is dominance relation in set A.



Holistic preference information

- Psychologists confirm that DMs are more confident exercising their decisions than explaining them.
- The most natural is a holistic pairwise comparison of some actions relatively well known to the DM, i.e. reference actions.



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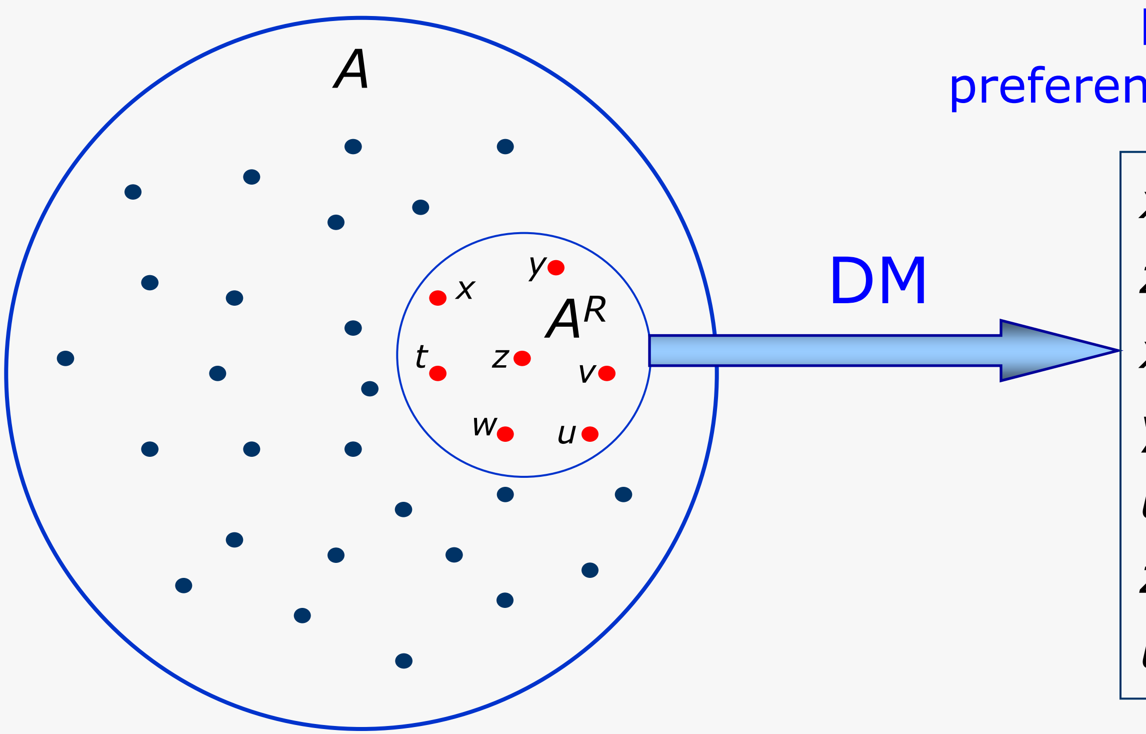
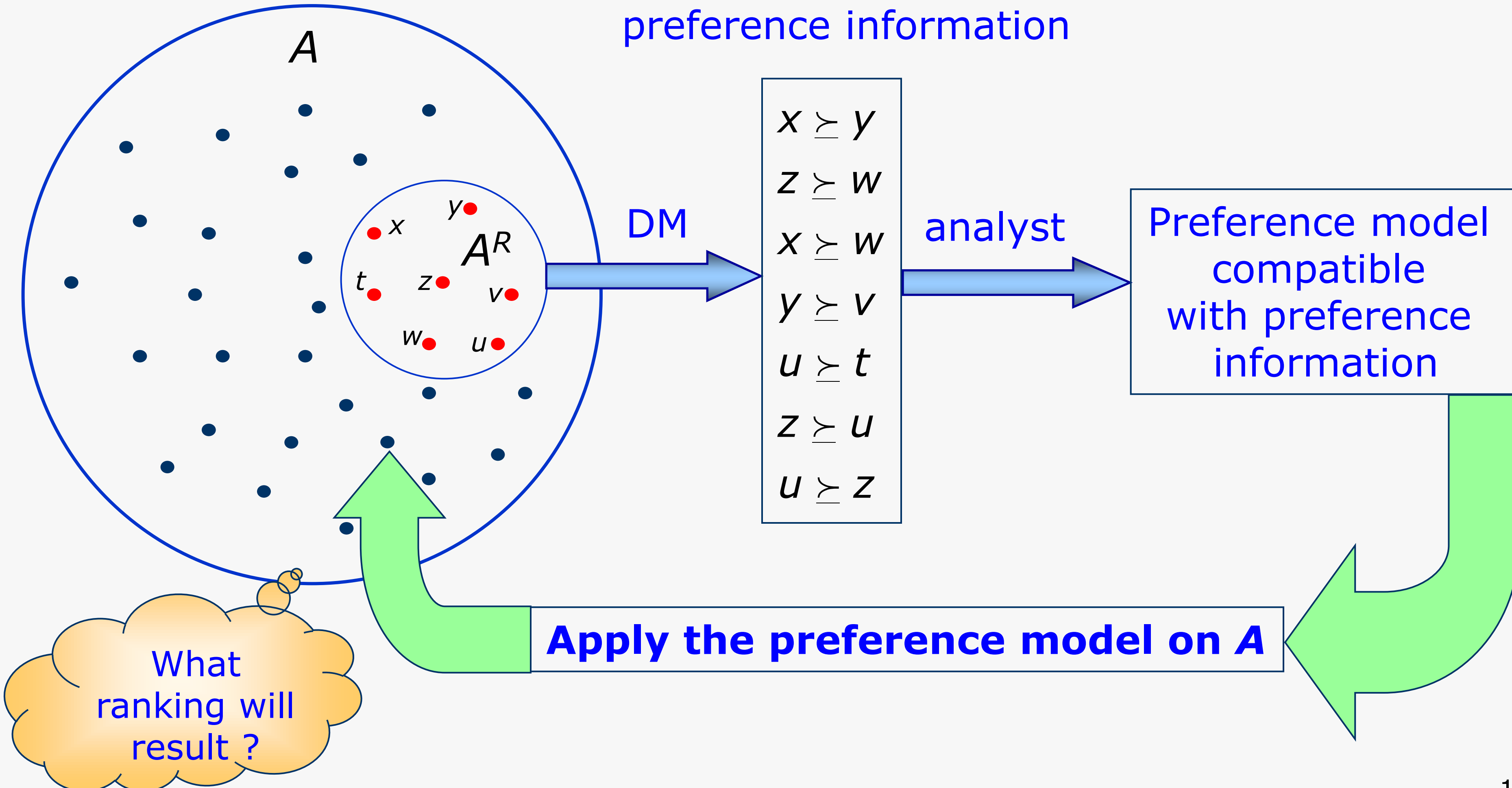


$x \succeq y$
$z \succeq w$
$x \succeq w$
$y \succeq v$
$u \succeq t$
$z \succeq u$
$u \succeq z$

Holistic preference information

- Question: Question: What is the consequence of using information gained in A^R for preference modeling on the whole set A?

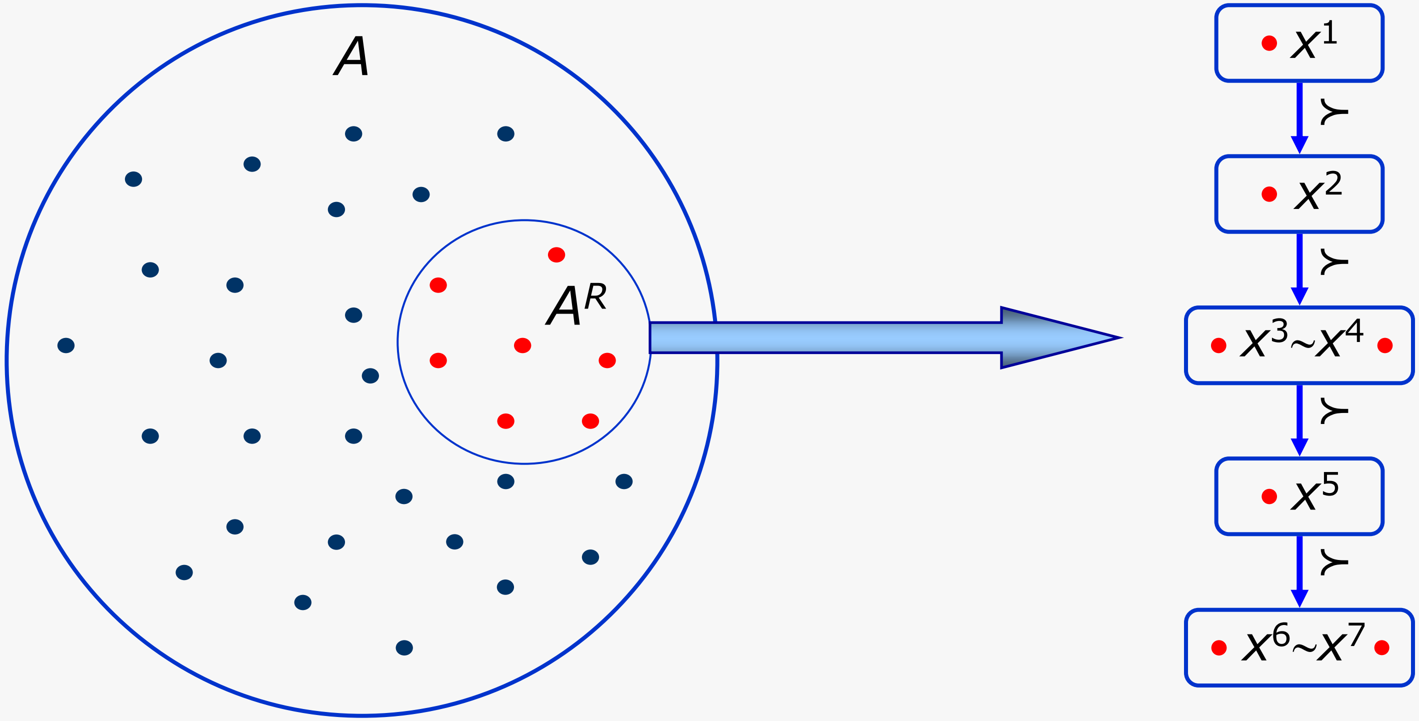


Principle of the ordinal regression

- The **preference information** is given in the form of a partial preorder on a subset of reference actions $A^R \subseteq A$.
- Additive value (or utility) function on A: for each $x \in A$

$$U(x) = \sum_{i=1}^{m} u_i[g_i(x)]$$

where u_i are non-decreasing marginal value functions



Robust ordinal regression

Basic question

- Remark 1:

  If there is one value function representing the preferences of the decision maker, in general, there are infinitely many others.

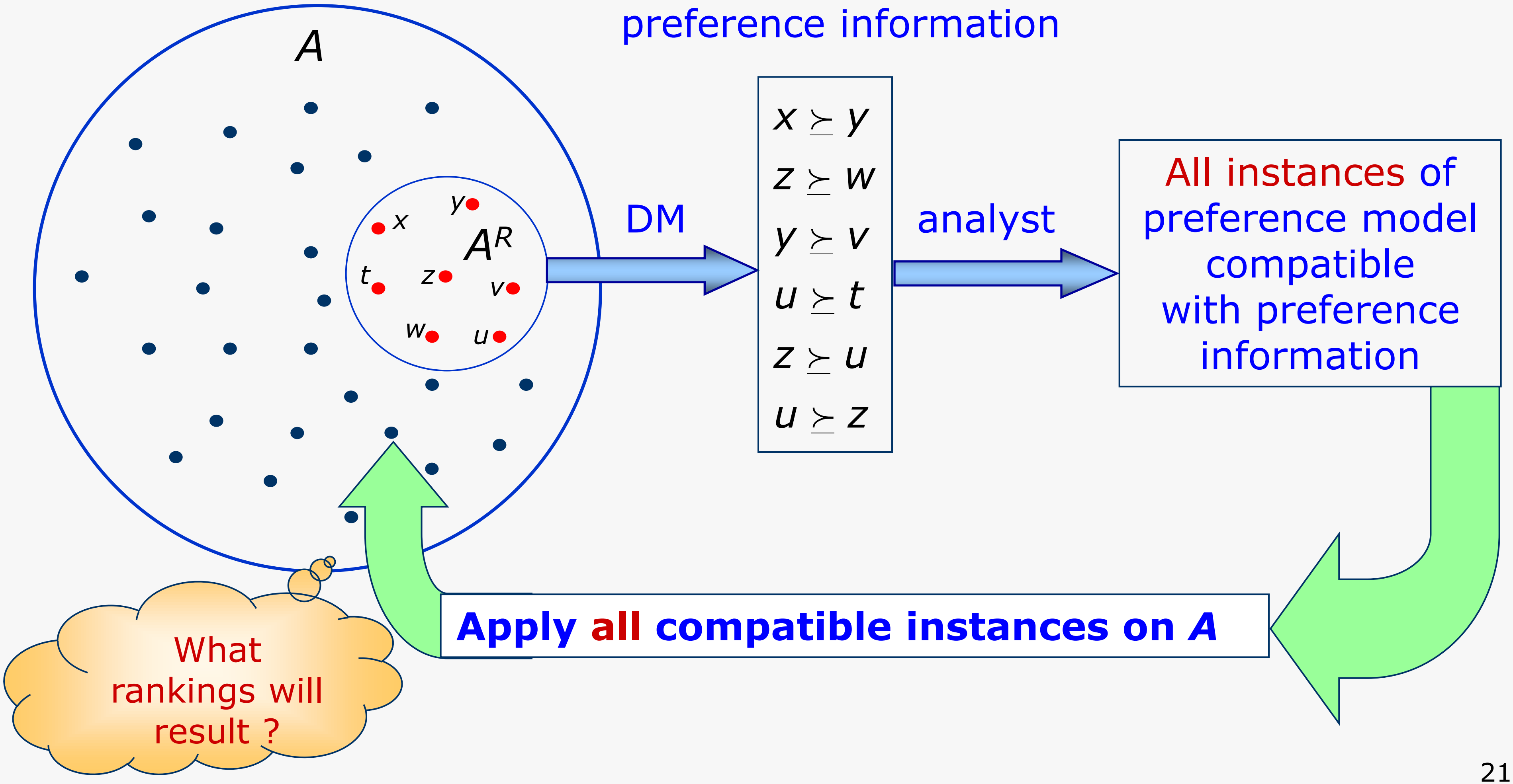
- Remark 2:

  In general, each one of these infinitely many value functions, gives a different ranking of actions from A.

- **Why to consider only one of these infinitely many value functions?**

One should use all compatible preference models on set A

- Question: what is the consequence of using all compatible preference models on set A ?



The UTA^{GMS} method

The UTA^{GMS} method (Greco, Mousseau and Słowiński 2004, 2008)

- DM is supposed to provide the following preference information:
  - a partial preorder $\succeq$ on A^R, such that $\forall x,y \in A^R$

$$x \succeq y \;\Leftrightarrow\; \text{„}x \text{ is at least as good as } y\text{”}$$

The UTA^{GMS} method (Greco, Mousseau and Słowiński 2004, 2008)

- A value function U is called compatible if it satisfies the constraints corresponding to DM's preference information:

a) $U(x) \geq U(y)$ iff $x \succeq y$

b) $U(x) > U(y)$ iff $x \succ y$

c) $U(x) = U(y)$ iff $x \sim y$

d) $u_i(x) \geq u_i(y)$ iff $x \succeq_i y, \; i \in I$

- Moreover, the following normalization constraints should also be taken into account:

e) $u_i(\alpha_i)=0, \; i \in I$

f) $\sum_{i \in I} u_i(\beta_i) = 1$

The UTA^{GMS} method (Greco, Mousseau and Słowiński 2004, 2008)

- If constraints *a) – f)* are consistent, then we get the two weak preference relations $\succeq^N$ and $\succeq^P$:
  - the necessary weak preference relation: for all $x,y \in A$, $x \succeq^N y \Leftrightarrow U(x) \geq U(y)$ for all compatible value functions (i.e. for all compatible value functions x is at least as good as y).
  - the possible weak preference relation: for all $x,y \in A$, $x \succeq^P y \Leftrightarrow U(x) \geq U(y)$ for at least one compatible value function (i.e. for at least one compatible value function x is at least as good as y).

From UTA^{GMS}–INT to UTA^{GSS}

Interactions between criteria in UTA^{GMS}-INT

- Positive interactions (example Mathematics and Literature):

$$U_{i_1,i_2}\left(g_{i_1}\left(x_{i_1}\right), g_{i_2}\left(x_{i_2}\right)\right) > U_{i_1}\left(g_{i_1}\left(x_{i_1}\right)\right) + U_{i_2}\left(g_{i_2}\left(x_{i_2}\right)\right)$$

- Negative interactions (example Mathematics and Physics):

$$U_{i_1,i_2}\left(g_{i_1}\left(x_{i_1}\right), g_{i_2}\left(x_{i_2}\right)\right) < U_{i_1}\left(g_{i_1}\left(x_{i_1}\right)\right) + U_{i_2}\left(g_{i_2}\left(x_{i_2}\right)\right)$$

- $F^{(2)} = \{\{g_{i1}, g_{i2}\}: g_{i1}, g_{i2} \in F\}$
- $Syn^{+} \subseteq F^{(2)}$, set of couples of criteria for which there is a positive synergy
- $Syn^{-} \subseteq F^{(2)}$, set of couples of criteria for which there is a negative synergy
- $Syn^{-} \cap Syn^{+} = \varnothing$
- Synergy strength: function $syn_{i1,i2}$: $X_{i1} \times X_{i2} \rightarrow [0,1]$, not decreasing in both arguments

The enriched additive value function in UTA^{GMS}-INT

- **UTA^{GMS}-INT** consider a value function of the type:

$$U^{\text{int}}(x) = \sum_{i=1}^{n} U_i\left[g_i(x)\right]$$

$$+ \sum_{\{g_{i_1}, g_{i_2}\} \in \text{Syn}^+} \text{syn}_{i_1, i_2}\left(g_{i_1}(x), g_{i_2}(x)\right) - \sum_{\{g_{i_1}, g_{i_2}\} \in \text{Syn}^-} \text{syn}_{i_1, i_2}\left(g_{i_1}(x), g_{i_2}(x)\right)$$

Interactions between criteria in UTA^{GSS}

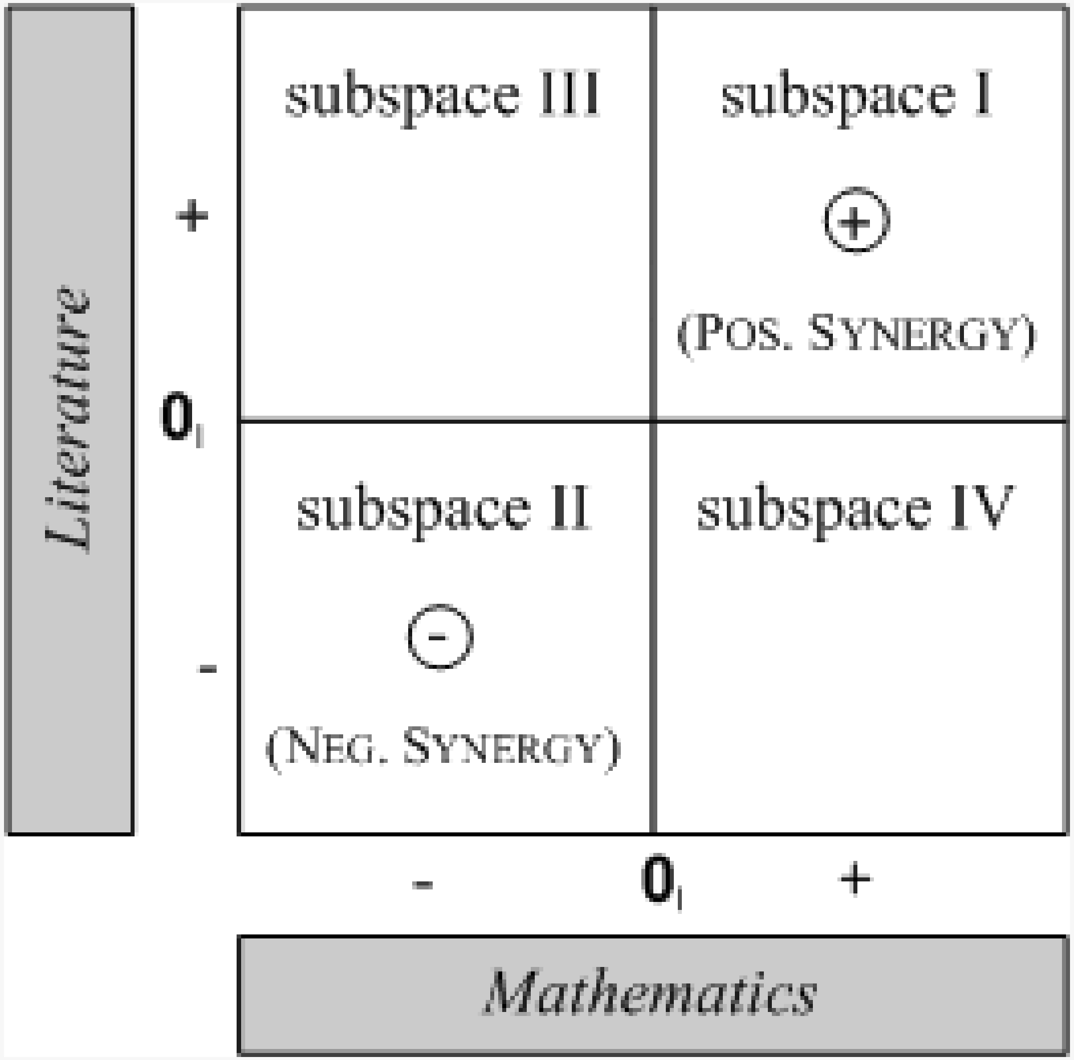
- Interactions depend on the sign of the evaluations with respect to neutral level.
- For example, consider scores in **Mathematics** and **Literature**.
- Positive interactions **for scores over the neutral levels** (good students in Mathematics are also good in Literature and therefore there is a positive synergy):

$$U_{i_1,i_2}\left(g_{i_1}\left(x_{i_1}\right),g_{i_2}\left(x_{i_2}\right)\right)>U_{i_1}\left(g_{i_1}\left(x_{i_1}\right)\right)+U_{i_2}\left(g_{i_2}\left(x_{i_2}\right)\right)$$

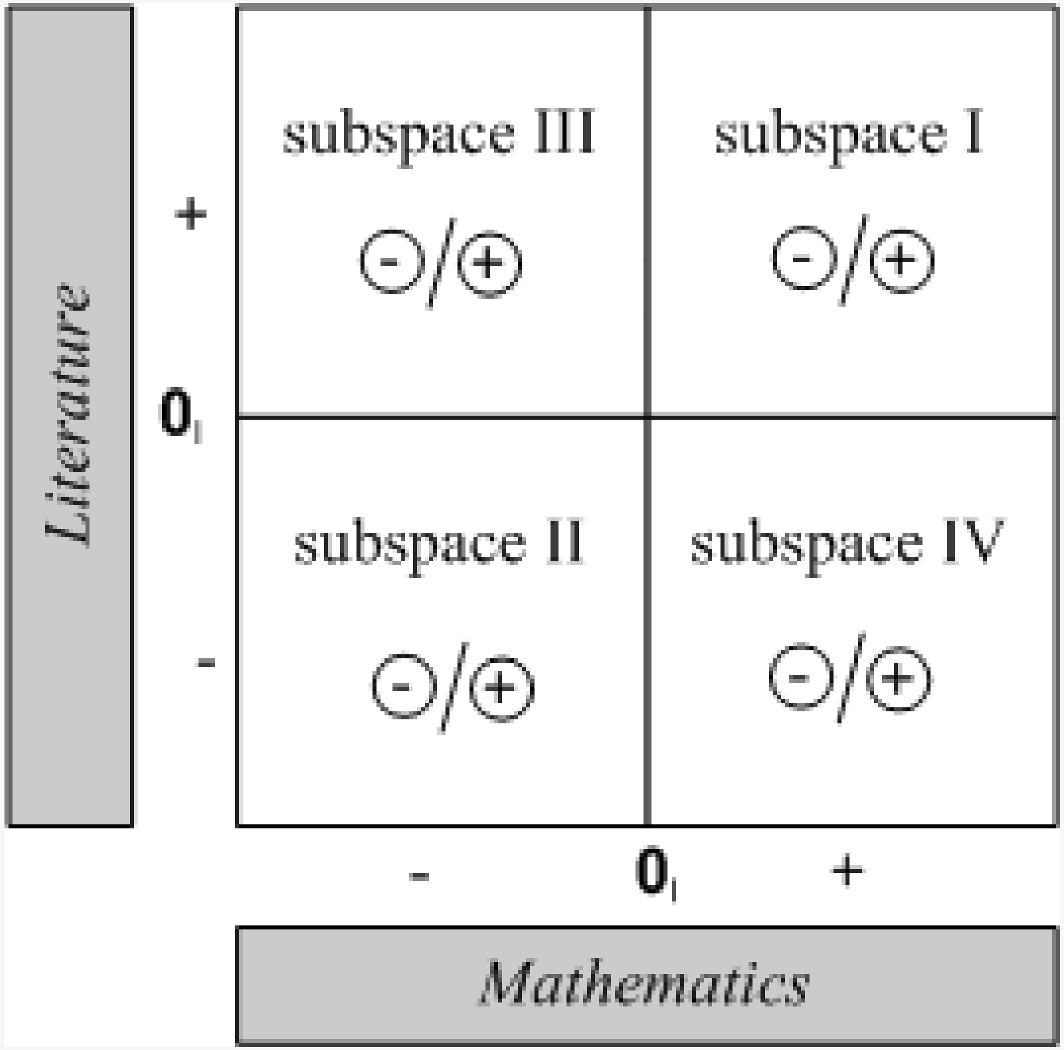
- Negative interactions **for scores under the neutral levels** (if a student is bad in Mathematics and in Literature the dean is not going to choose him, independent of his score in Physics. There is a negative synergy):

$$U_{i_1,i_2}\left(g_{i_1}\left(x_{i_1}\right),g_{i_2}\left(x_{i_2}\right)\right)<U_{i_1}\left(g_{i_1}\left(x_{i_1}\right)\right)+U_{i_2}\left(g_{i_2}\left(x_{i_2}\right)\right)$$

Interactions between criteria in UTA^{GSS}



Interactions between criteria in UTA^{GSS}



Interactions between criteria in UTA^{GSS}

- $F^{(2)} = \{\{g_{i1}, g_{i2}\}: g_{i1}, g_{i2} \in F\}$.
- $Syn^{++,+} \subseteq F^{(2)}$, set of couples of criteria for which there is a positive synergy in case of evaluations over the neutral level,
- $Syn^{++,-} \subseteq F^{(2)}$, set of couples of criteria for which there is a negative synergy in case of evaluations over the neutral level,
- $Syn^{-+,+} \subseteq F^2$, set of pairs of criteria for which there is a negative synergy in case of evaluation under the neutral level for the first criterion and over the neutral level for the second criterion,
- $Syn^{-+,-} \subseteq F^2$, set of pairs of criteria for which there is a negative synergy in case of evaluation under the neutral level for the first criterion and over the neutral level for the second criterion,
- $Syn^{--,+} \subseteq F^{(2)}$, set of couples of criteria for which there is a positive synergy in case of evaluations under the neutral level,
- $Syn^{--,-} \subseteq F^{(2)}$, set of couples of criteria for which there is a negative synergy in case of evaluations under the neutral level.

Interactions between criteria in UTA^{GSS}

- Observe that

$$Syn^{++,+} \cap Syn^{++,-} = \emptyset, \; Syn^{-+,+} \cap Syn^{-+,-} = \emptyset, \; Syn^{--,+} \cap Syn^{--,-} = \emptyset.$$

- However, not necessarily

$$Syn^{++,-} \cap Syn^{--,+} = \emptyset.$$

- In fact, for example, between scores in Mathematics and Literature we can have a positive synergy in case of scores over the neutral level, i.e.

$$\{\text{Mathematics, Literature}\} \in Syn^{++,+},$$

but also a negative synergy in case of scores under the neutral level, i.e.

$$\{\text{Mathematics, Literature}\} \in Syn^{--,-}.$$

Thus,

$$\{\text{Mathematics, Literature}\} \in Syn^{++,-} \cap Syn^{--,+} \neq \emptyset.$$

- Analogously, not necessarily

$$Syn^{++,-} \cap Syn^{--,+} = \emptyset, \; Syn^{++,-} \cap Syn^{-+,+} = \emptyset, \text{ etc.}$$

The enriched additive value function in UTA^{GSS}

- We consider a value function of the type:

$$U^{\text{int}}(x) = \sum_{i=1}^{n} U_i\left[g_i(x)\right]$$

$$+ \sum_{\{g_{i_1},g_{i_2}\}\in \text{Syn}^{++,+}} \text{syn}_{i_1,i_2}^{++,+}\left(g_{i_1}(x),g_{i_2}(x)\right) - \sum_{\{g_{i_1},g_{i_2}\}\in \text{Syn}^{++,-}} \text{syn}_{i_1,i_2}^{++,-}\left(g_{i_1}(x),g_{i_2}(x)\right)$$

$$+ \sum_{(g_{i_1},g_{i_2})\in \text{Syn}^{-+,+}} \text{syn}_{i_1,i_2}^{-+,+}\left(g_{i_1}(x),g_{i_2}(x)\right) - \sum_{(g_{i_1},g_{i_2})\in \text{Syn}^{+-,-}} \text{syn}_{i_1,i_2}^{-+,-}\left(g_{i_1}(x),g_{i_2}(x)\right)$$

$$+ \sum_{\{g_{i_1},g_{i_2}\}\in \text{Syn}^{--,+}} \text{syn}_{i_1,i_2}^{--,+}\left(g_{i_1}(x),g_{i_2}(x)\right) - \sum_{\{g_{i_1},g_{i_2}\}\in \text{Syn}^{--,-}} \text{syn}_{i_1,i_2}^{--,-}\left(g_{i_1}(x),g_{i_2}(x)\right)$$

Compatible value functions

$$U^{\text{int}}(x) \geq U^{\text{int}}(y) \text{ if } x \succ y$$

$$U^{\text{int}}(x) = U^{\text{int}}(y) \text{ if } x \sim y$$

$$U^{\text{int}}(x) - U^{\text{int}}(y) \geq U^{\text{int}}(w) - U^{\text{int}}(z) \quad \text{if } (x,y) \succsim^{*} (w,z)$$

$$u_i(g_i(x)) - u_i(g_i(y)) \geq u_i(g_i(w)) - u_i(g_i(z)) \quad \text{if } (x,y) \succsim_i^{*} (w,z), \quad i = 1,\ldots,n,$$

$$U^{\text{int}}(x) \geq U^{\text{int}}(y) \text{ if } g_i(x) \geq g_i(y) \quad \text{for all } i = 1,\ldots,n,$$

$$u_i\left(g_i\left(x_{\tau_i(j)}\right)\right) - u_i\left(g_i\left(x_{\tau_i(j-1)}\right)\right) \geq 0,\ i = 1,\ldots,n, \quad j = 2,\ldots,m,$$

$$\text{syn}_{i_1,i_2}^{\sigma_1\sigma_2,\sigma_3}\left(\omega_{i_1}, \omega_{i_2}\right) = 0, \quad i_1, i_2 = 1,\ldots,n,\ \sigma_1, \sigma_2, \sigma_3 \in \{-,+\},$$

$$U^{\text{int}}(x) = 0 \quad \text{if } g_i(x) = \alpha_i,\ i = 1,\ldots,n,$$

$$U^{\text{int}}(x) = 1 \quad \text{if } g_i(x) = \beta_i,\ i = 1,\ldots,n.$$

Monotocity and boundary conditions of the synergy function

$$\mathrm{syn}_{i_1,i_2}^{++,+}\left(g_{i_1}(x),g_{i_2}(x)\right) \text{ and } \mathrm{syn}_{i_1,i_2}^{++,-}\left(g_{i_1}(x),g_{i_2}(x)\right)$$

Non-decreasing in both of the two arguments

$$\mathrm{syn}_{i_1,i_2}^{-+,+}\left(g_{i_1}(x),g_{i_2}(x)\right) \text{ and } \mathrm{syn}_{i_1,i_2}^{-+,-}\left(g_{i_1}(x),g_{i_2}(x)\right)$$

Non-increasing in the first argument and non-decreasing in the second argument

$$\mathrm{syn}_{i_1,i_2}^{--,+}\left(g_{i_1}(x),g_{i_2}(x)\right) \text{ and } \mathrm{syn}_{i_1,i_2}^{--,-}\left(g_{i_1}(x),g_{i_2}(x)\right)$$

Non-increasing in both of the two arguments

Two possibilities for determinining pairs of interacting criteria

- **1)** The DM explicitly says which pairs of criteria are interacting and in which form, i.e. he or she gives $Syn^{++,+}$, $Syn^{++,-}$, $Syn^{--,+}$, $Syn^{--,-}$, $Syn^{-+,+}$, $Syn^{-+,-}$.
- **2)** The DM does not say which pairs of criteria are interacting and in which form, i.e. he or she does not give $Syn^{++,+}$, $Syn^{++,-}$, $Syn^{--,+}$, $Syn^{--,-}$, $Syn^{-+,+}$, $Syn^{-+,-}$.
- In case 2) sets of pairs of interacting criteria with specific form of interactions that explain the DM preferences can be “discovered” with a proper mixed integer linear programming model.
- Case 1) and 2) are not alternative, but complementary:
  - the DM can give the pairs of interacting criteria but he can also be interested in see if there are other possible sets of pairs of interacting criteria with specific form of interactions.
  - In any case the DM has to select one set of pairs of interacting criteria among the plurality of such sets given by the model.

The mixed integer LP model to determine sets of pairs of interacting criteria

- We consider the following constraints:

$$\mathrm{syn}_{i_1,i_2}^{++,+}\left(g_{i_1}(x),g_{i_2}(x)\right)\le \delta_{i_1,i_2}^{++,+},\ \mathrm{syn}_{i_1,i_2}^{++,-}\left(g_{i_1}(x),g_{i_2}(x)\right)\le \delta_{i_1,i_2}^{++,-},\forall\{g_{i_1},g_{i_2}\}\in F^{(2)},$$

$$\mathrm{syn}_{i_1,i_2}^{-+,+}\left(g_{i_1}(x),g_{i_2}(x)\right)\le \delta_{i_1,i_2}^{-+,+},\mathrm{syn}_{i_1,i_2}^{-+,-}\left(g_{i_1}(x),g_{i_2}(x)\right)\le \delta_{i_1,i_2}^{-+,-},\forall(g_{i_1},g_{i_2})\in F^{2},$$

$$\mathrm{syn}_{i_1,i_2}^{--,+}\left(g_{i_1}(x),g_{i_2}(x)\right)\le \delta_{i_1,i_2}^{--,+},\ \mathrm{syn}_{i_1,i_2}^{--,-}\left(g_{i_1}(x),g_{i_2}(x)\right)\le \delta_{i_1,i_2}^{--,-},\forall\{g_{i_1},g_{i_2}\}\in F^{(2)},$$

$$\delta_{i_1,i_2}^{++,+},\delta_{i_1,i_2}^{++,-},\delta_{i_1,i_2}^{-+,+},\delta_{i_1,i_2}^{-+,-},\delta_{i_1,i_2}^{--,+},\delta_{i_1,i_2}^{--,-}\in\{0,1\}.$$

- Using the above constraints we minimize the following sum:

$$\sum_{\{g_{i_1},g_{i_2}\}\in F^{(2)}}\delta_{i_1,i_2}^{++,+}+\sum_{\{g_{i_1},g_{i_2}\}\in F^{(2)}}\delta_{i_1,i_2}^{++,-}+\sum_{(g_{i_1},g_{i_2})\in F^{2}}\delta_{i_1,i_2}^{-+,+}+$$

$$+\sum_{(g_{i_1},g_{i_2})\in F^{2}}\delta_{i_1,i_2}^{-+,-}+\sum_{\{g_{i_1},g_{i_2}\}\in F^{(2)}}\delta_{i_1,i_2}^{--,+}+\sum_{\{g_{i_1},g_{i_2}\}\in F^{(2)}}\delta_{i_1,i_2}^{--,-}.$$

Example for a Knock out Criterion

- The dean strongly prefers students who are „not bad“ in Mathematics (i_1) as well as in Physics (i_2) in comparison to those who are.
- This preference could represented with the help of two pairwise comparisons:

I $\quad m_1 + b_2 + b_3 > b_1 + b_2 + g_3$

II $\quad b_1 + m_2 + g_3 > g_1 + g_2 + m_3$

For simplification we substract all smaller Performances:

Ia $\quad (m_1 - b_1) > (g_3 - b_3)$

IIa $\quad (g_3 - m_3) > (g_1 - b_1) + (g_2 - m_2)$

Ia,IIa in III:

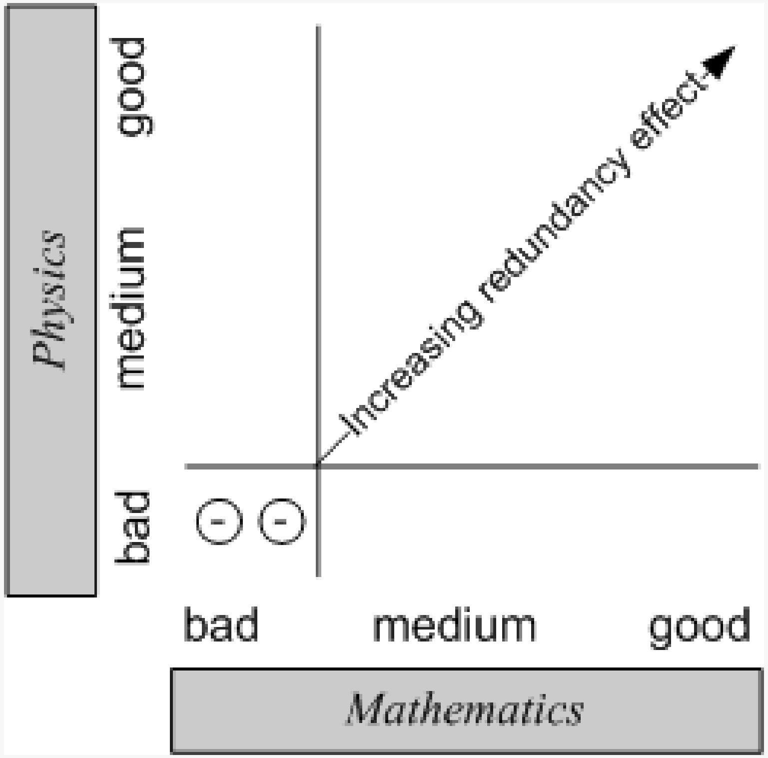
$$(m_1 - b_1) > (g_3 - b_3) \geq (g_3 - m_3) > (g_1 - b_1) + (g_2 - m_2)$$

$$(m_1 - b_1) > \cancel{(g_3 - b_3)} \geq \cancel{(g_3 - m_3)} > (g_1 - b_1) + \cancel{(g_2 - m_2)}$$

$(m_1 - b_1) > (g_1 - b_1)$ cannot be true without an interaction!

Illustrative example

Example for an interaction which only can be modeled with UTA^{GSS}



- Redundancy for students who are medium or better in both subjects.
- Knock out criterion for students who are bad in both subjects.

Considered students

Students	Mathematics	Physics	Literature
S1	Good	Medium	Bad
S2	Medium	Medium	Medium
S3	Good	Bad	Bad
S4	Medium	Bad	Medium
S5	Bad	Medium	Good
S6	Medium	Bad	Good
S7	Good	Good	Medium
S8	Bad	Medium	Bad
S9	Medium	Bad	Bad
S10	Bad	Bad	Good

Preference information given by the DM

- **Preferences between students**

$S1 \succ S2$

$S3 \succ S4$

Redundance

$S5 \succ S7$

$S6 \succ S7$

$S8 \succ S10$

$S9 \succ S10$

Knock out criterion

- **Overall intensity of criteria**

$(\text{Good}, \text{Bad})_{\text{Lit}} \succ^{*} (\text{Good}, \text{Bad})_{\text{Math}}$

$(\text{Good}, \text{Bad})_{\text{Lit}} \succ^{*} (\text{Good}, \text{Bad})_{\text{Phys}}$

Illustrative example: UTA^{GMS}-INT (**Synergy** between Math & Phys.)

	Mathematics	Physics	Literature
Good	0.27	0.19	0.35
Medium	0.23	0.19	0.06
Bad	0	0	0

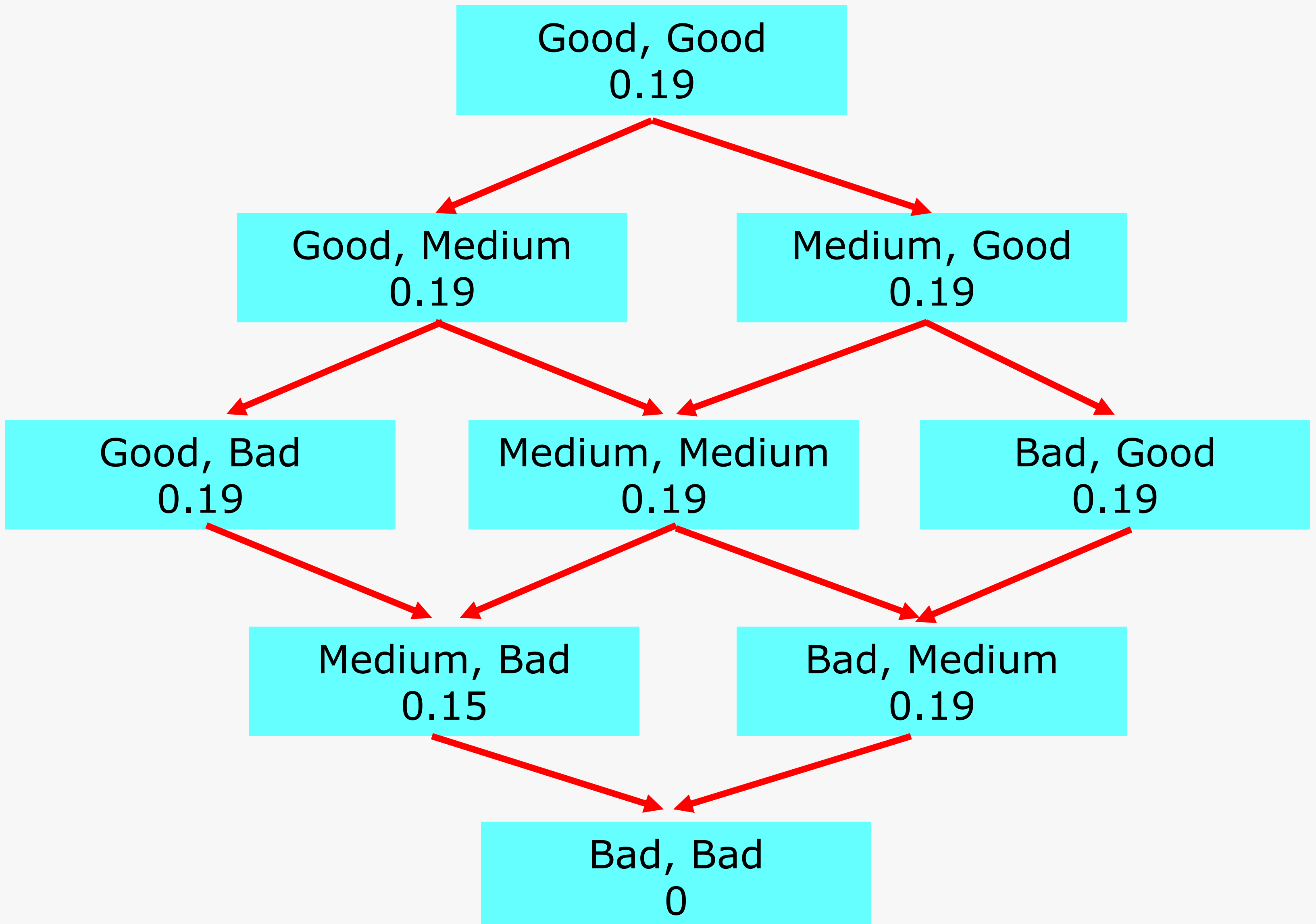
$\varepsilon=0.02083$

Illustrative example: UTA^{GMS}-INT (**Synergy** between Math & Phys.)

Students	**Mathe-matics**	**Physics**	**Literature**	**Synergy (Math., Phys.)**	**Global Score**
S1	Good 0.27	Medium 0.19	Bad 0	Good,Medium 0.19	**0,65**
S2	Medium 0.23	Medium 0.19	Medium 0.06	Medium,Medium 0.19	**0,67**
S3	Good 0.27	Bad 0	Bad 0	Good,Bad 0.19	**0,46**
S4	Medium 0.23	Bad 0	Medium 0.06	Medium,Bad 0.15	**0,44**
S5	Bad 0	Medium 0.19	Good 0.35	Bad,Medium 0.19	**0,73**
S6	Medium 0.23	Bad 0	Good 0.35	Medium,Bad 0.15	**0,73**
S7	Good 0.27	Good 0.19	Medium 0.06	Good,Good 0.19	**0,71**
S8	Bad 0	Medium 0.19	Bad 0	Bad,Medium 0.19	**0,38**
S9	Medium 0.23	Bad 0	Bad 0	Medium,Bad 0.15	**0,38**
S10	Bad 0	Bad 0	Good 0.35	**Bad,Bad 0**	**0,35**

$\varepsilon=0.020803$ S1$\succ$S2, S3$\succ$S4, S5$\succ$S7, S6$\succ$S7, S8$\succ$S10, S9$\succ$S10

Illustrative example: UTA^{GMS}-INT (**Synergy** between Math & Phys.)



Illustrative example: UTA^{GMS}-INT (**Redundancy** betw. Math & Phys.)

	Mathematics	Physics	Literature
Good	0	0	0
Medium	0	0	0
Bad	0	0	0

No solution! Epsilon= 0

Illustrative example: UTA^{GSS} (**Bipolar redundancy and Knock out criterion** between Math & Phys.)

	Mathematics	Physics	Literature
Good	0.36	0.63	0.36
Medium	0.18	0.45	0.09
Bad	0	0.27	0

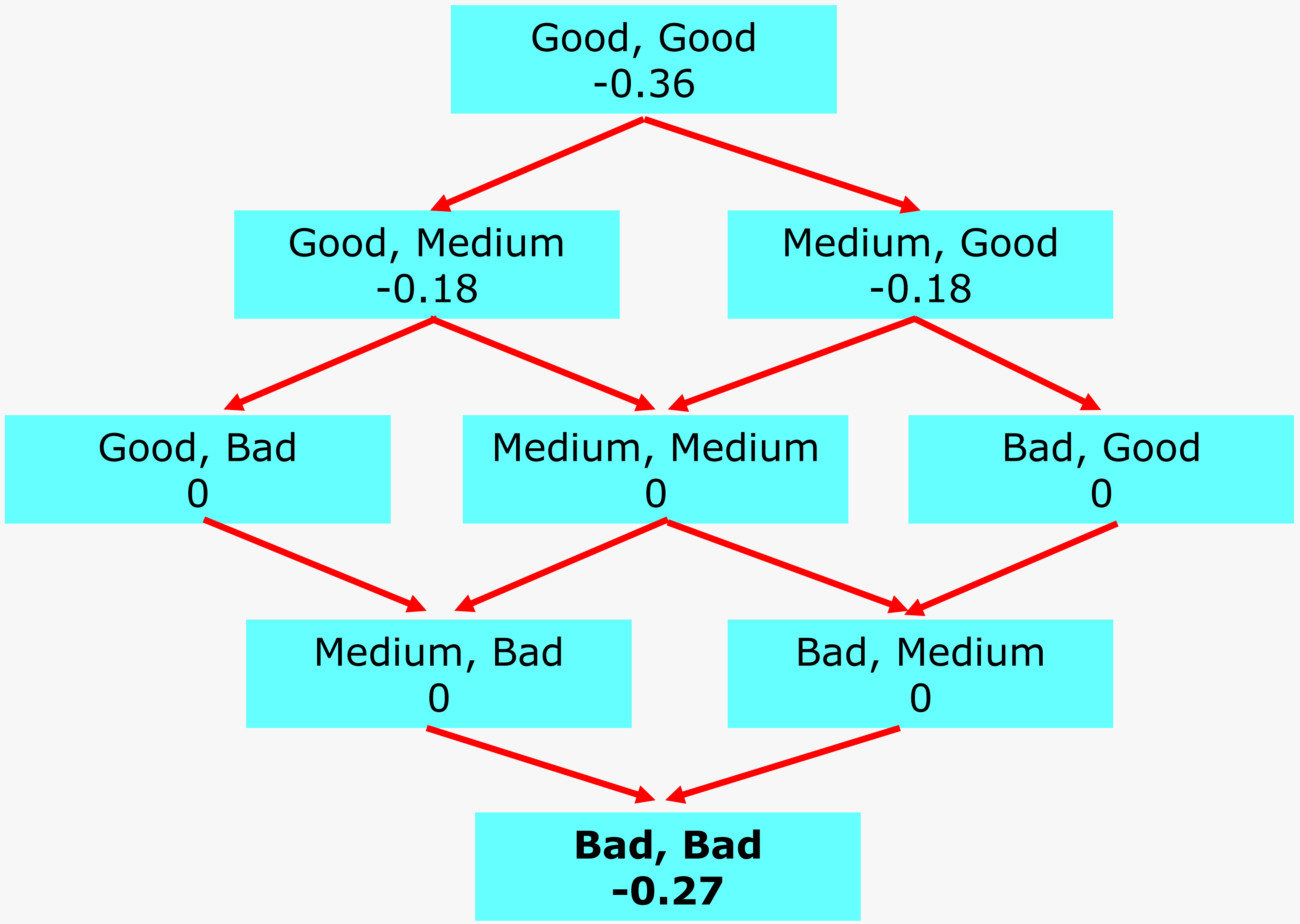
$\varepsilon=0.0909$

Illustrative example: UTA^{GSS} (**Bipolar redundancy and Knock out criterion** between Math & Phys.)

Students	Mathe-matics	Physics	Literature	Synergy (Math., Phys.)	Global Score
S1	Good 0.36	Medium 0.45	Bad 0	Good,Medium -0.18	**0,64**
S2	Medium 0.18	Medium 0.45	Medium 0.09	Medium,Medium 0	**0,73**
S3	Good 0.36	Bad 0.27	Bad 0	Good,Bad 0	**0,64**
S4	Medium 0.18	Bad 0.27	Medium 0.09	Medium,Bad 0	**0,55**
S5	Bad 0	Medium 0.45	Good 0.36	Bad,Medium 0	**0,82**
S6	Medium 0.18	Bad 0.27	Good 0.36	Medium,Bad 0	**0,82**
S7	Good 0.36	Good 0.63	Medium 0.09	Good,Good -0.36	**0,73**
S8	Bad 0	Medium 0.45	Bad 0	Bad,Medium 0	**0,45**
S9	Medium 0.18	Bad 0.27	Bad 0	Medium,Bad 0	**0,45**
S10	Bad 0	Bad 0.27	Good 0.36	**Bad,Bad -0.27**	**0,36**

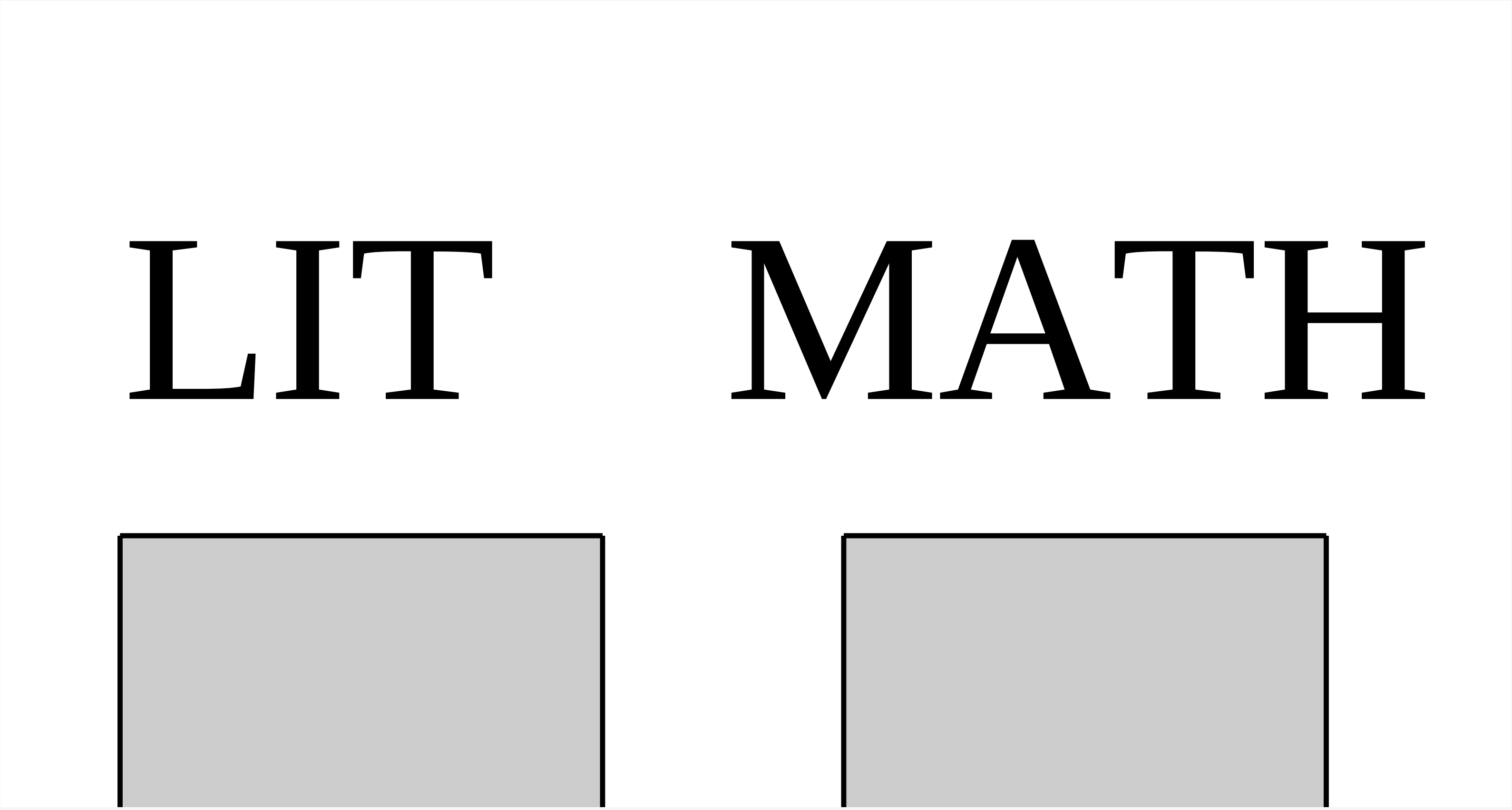
$\varepsilon=0.0909$ S1$\succ$S2, S3$\succ$S4, S5$\succ$S7, S6$\succ$S7, S8$\succ$S10, S9$\succ$S10

Illustrative example: UTA^{GSS} (**Bipolar redundancy and Knock out criterion** between Math & Phys.)

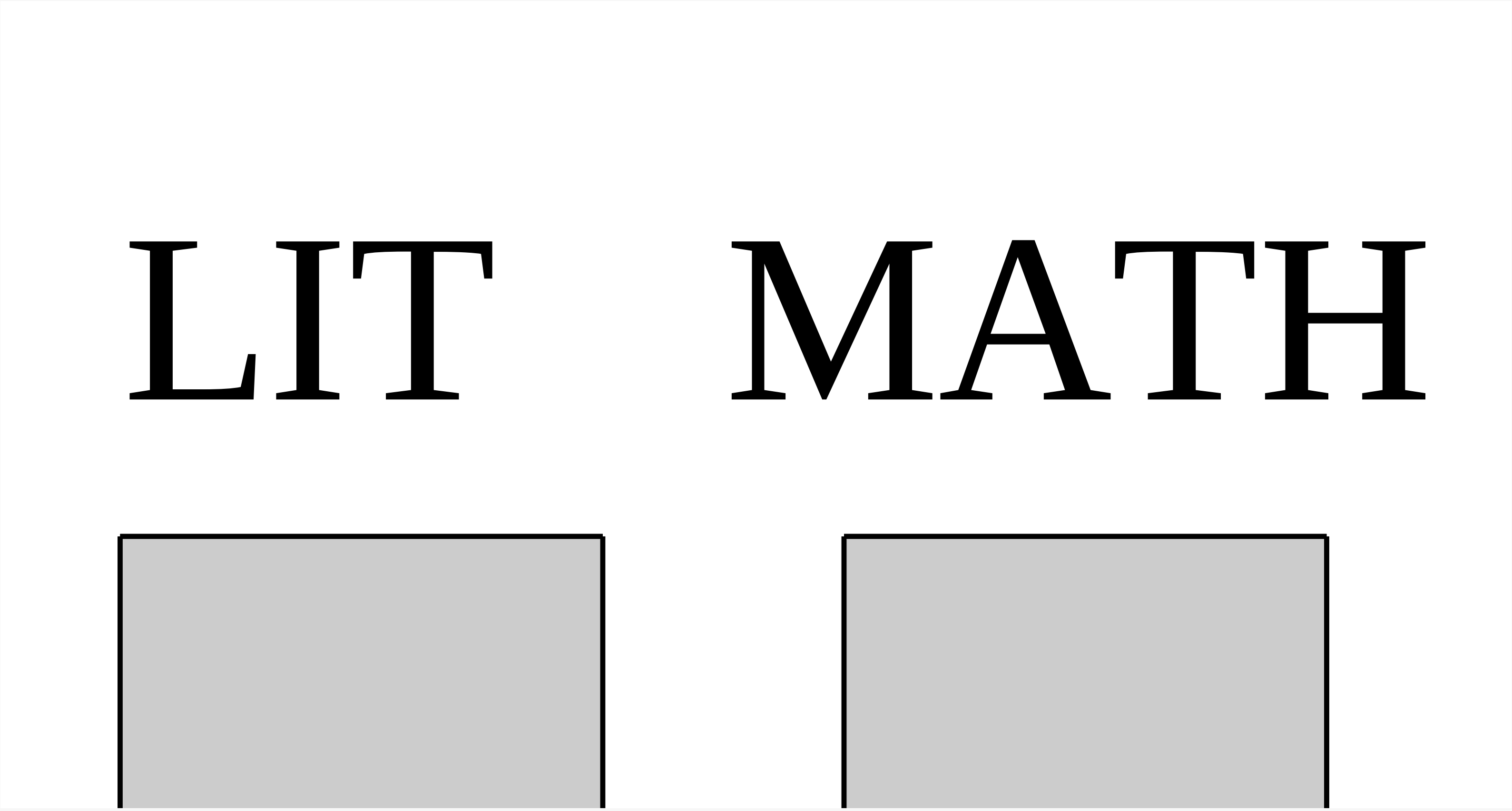


UTA^{GSS} vs. UTA^{GMS}-INT

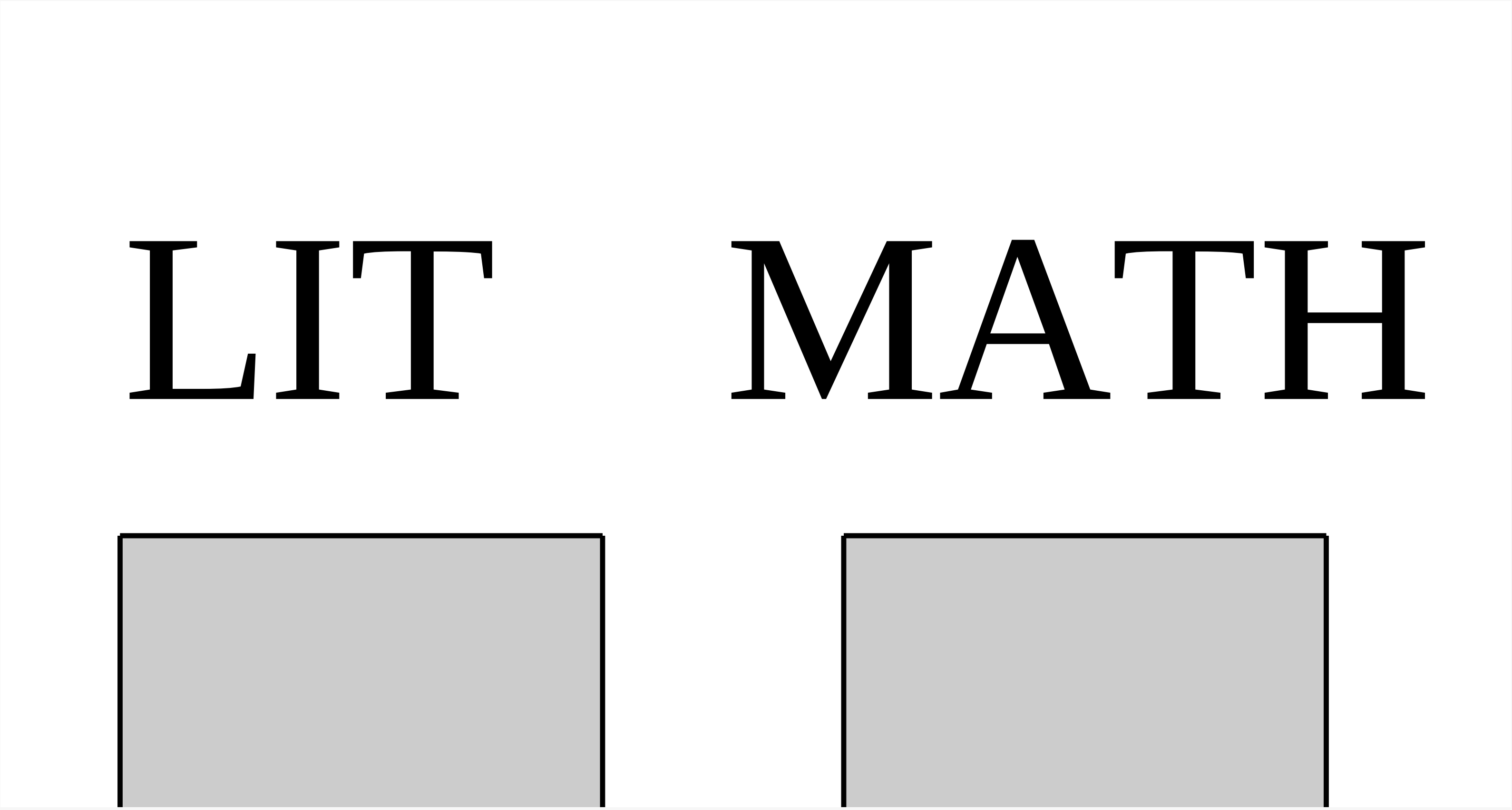
UTA^{GSS} a generalization of UTA^{GMS}-INT?



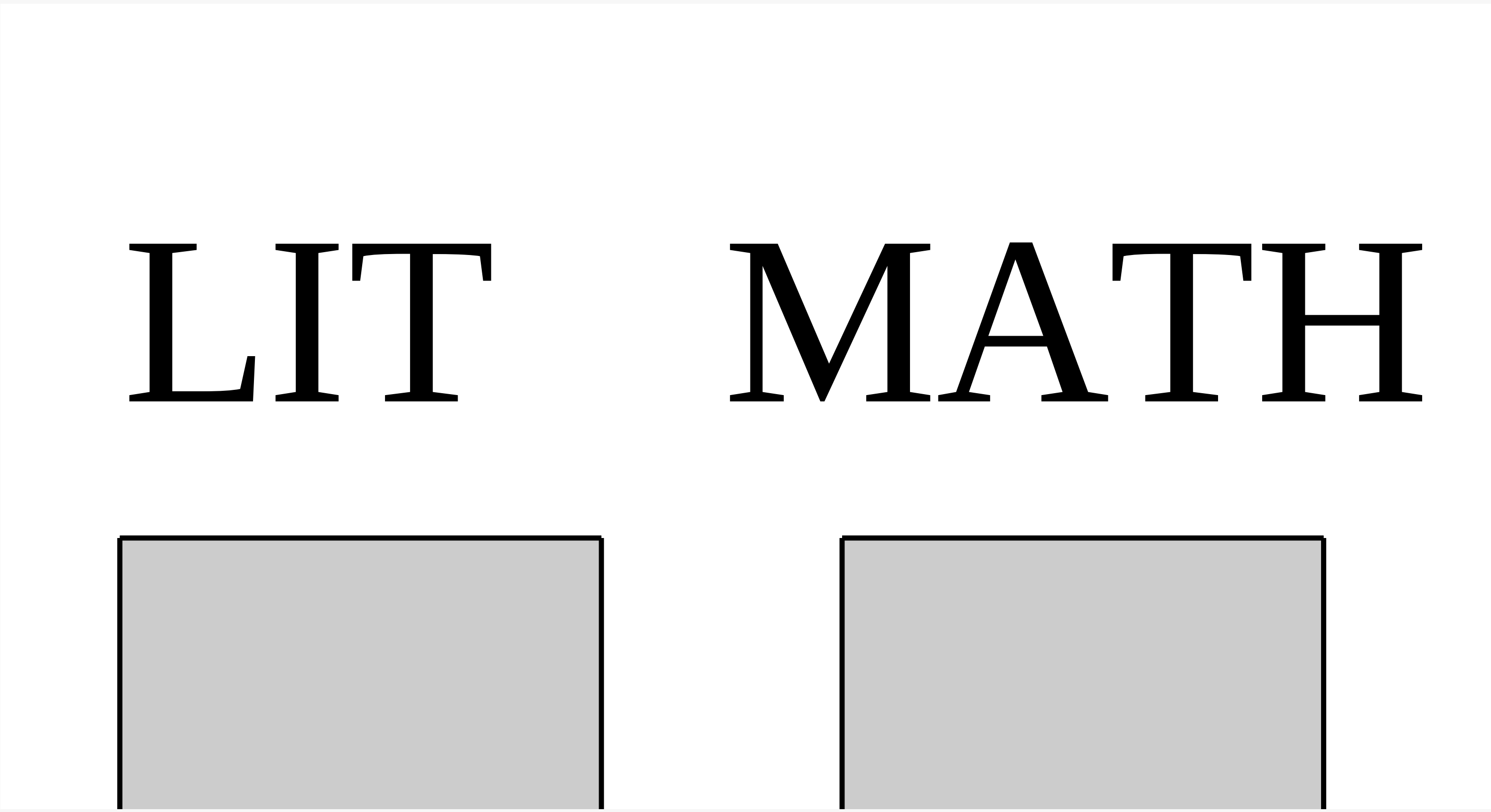
UTA^{GSS} a generalization of UTA^{GMS}-INT?



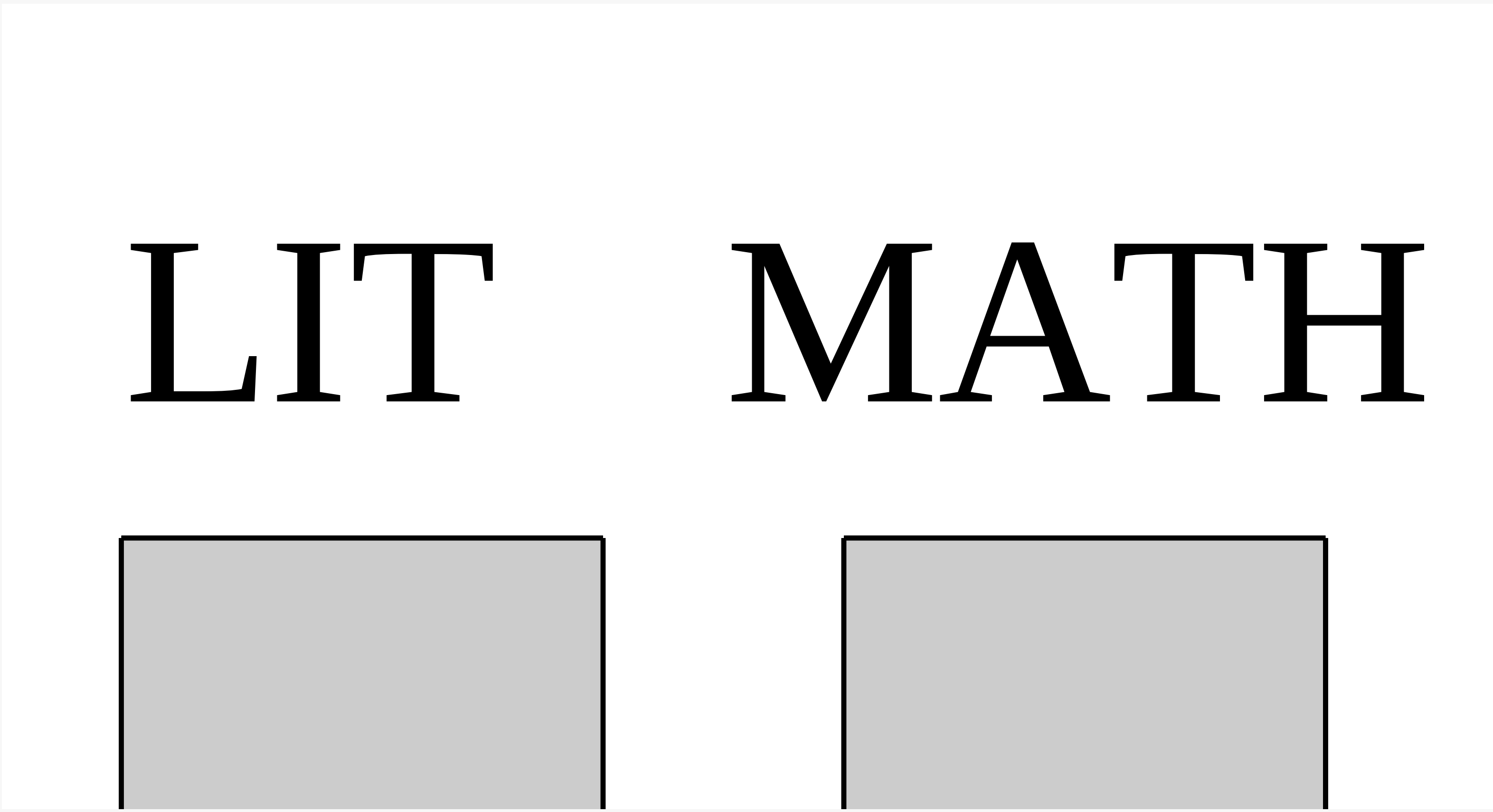
UTA^{GSS} a generalization of UTA^{GMS}-INT?



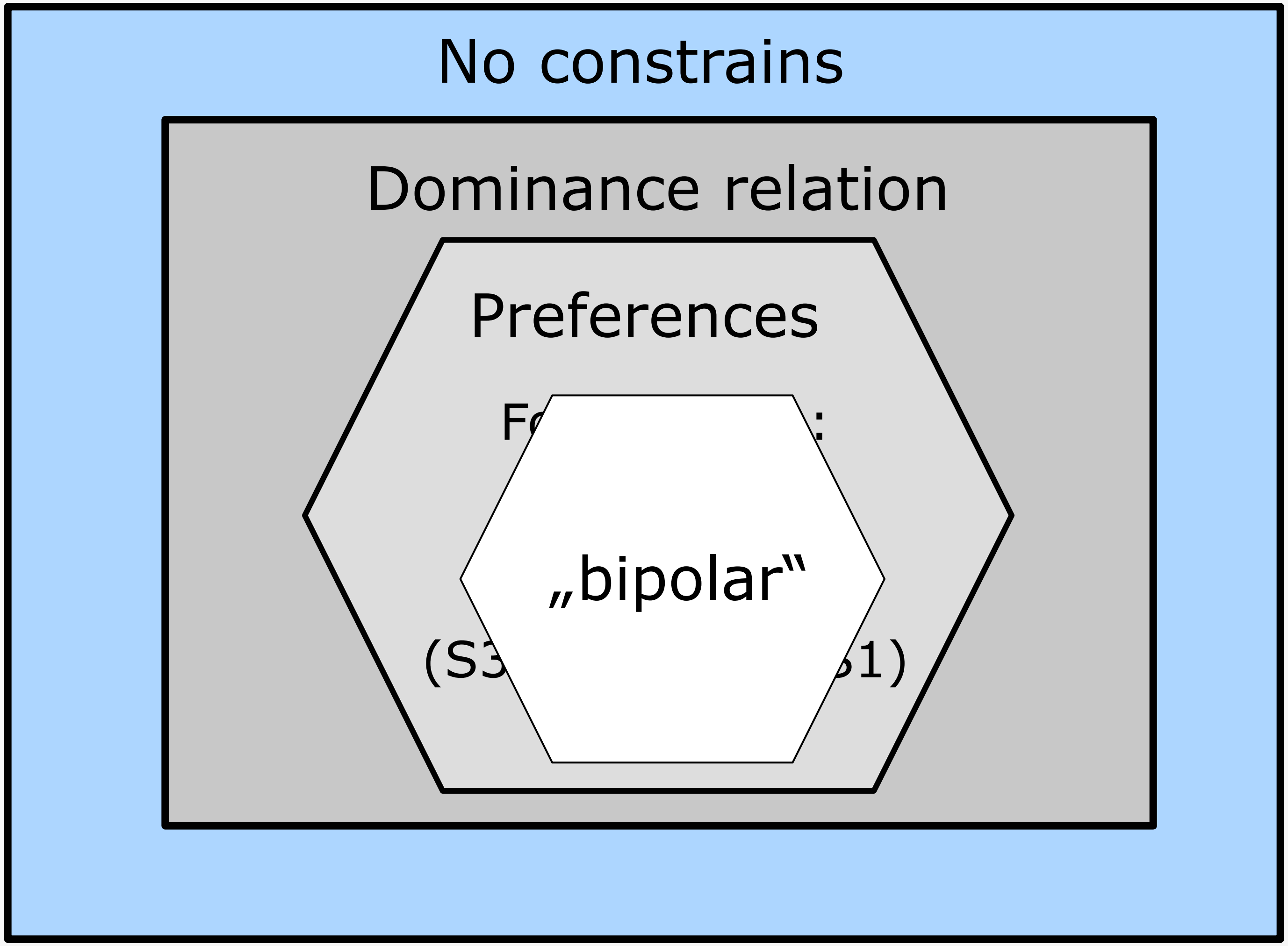
UTA^{GSS} a generalization of UTA^{GMS}-INT?



UTA^{GSS} a generalization of UTA^{GMS}-INT?



Compatible value functions under consideration of different restrictions



Conclusions

Conclusions

- We presented a robust ordinal regression method, **UTA^{GSS}**, which is able to deal with positive and negative synergy between criteria.
- **UTA^{GSS}** allows the computation of possible and necessary rankings and is able to calculate the most discriminative and the most representative value function.
- The methodology we proposed has **several advantages with respect to the UTA^{GMS}-INT**:
  - **UTA^{GSS}** is **more powerful** since it represents bipolar interactions **UTA^{GMS}-INT** is not able to represent;
  - **UTA^{GSS}** is a generalisation of **UTA^{GMS}-INT**.
- The same methodology can be extended straightforward to sorting problems, giving the **UTADIS^{GSS}** method.
- The same methodology can be extended to group decisions, originating the methods **UTA^{GSS}-GROUP** and **UTADIS^{GSS}-GROUP**.

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